Lesson 1.4 Exercises, pages 46–54

A

3. Which functions are polynomial functions? Justify your choices.

a) $f(x) = 2\sqrt{x} - x^2$

Not a polynomial function: $\sqrt{x} = x^{\frac{1}{2}}$ and $\frac{1}{2}$ is not a whole number.

b) $g(x) = 6x^3 - x^2 + 3x - 7$

Polynomial function: the coefficients of the variables are real numbers and all exponents are whole numbers.

c) $h(x) = 7x^2 + 2x^3 - x - \frac{1}{2}$

Polynomial function: the coefficients of the variables are real numbers and all exponents are whole numbers.

d) $k(x) = 3^x + 5$

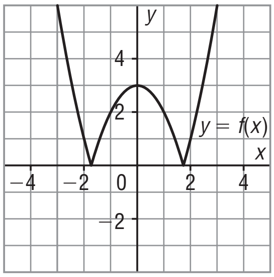
Not a polynomial function: the variable x is an exponent.

e) $p(x) = 5x^2 - 7x + \frac{2}{x}$

Not a polynomial function: $\frac{2}{x} = 2x^{-1}$ and the exponent is not a whole number.

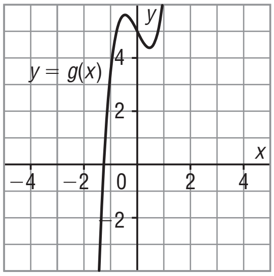
4. Which graphs are graphs of polynomial functions? Justify your answers.

a)



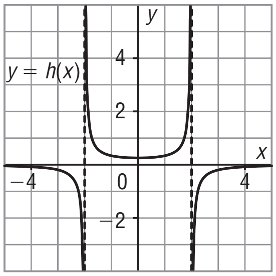
No, graph has sharp corners.

b)



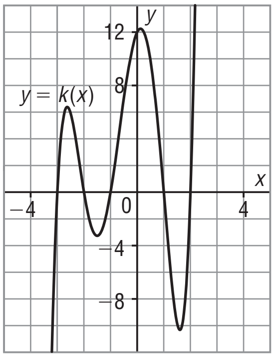
Yes, graph is smooth and continuous and has a possible shape for an odd-degree polynomial function.

c)



No, graph is not continuous.

d)



Yes, graph is smooth and continuous and has a possible shape for an odd-degree polynomial function.

5. Complete the table below. The first row has been done for you.

	Equation	Degree	Odd or Even Degree	Type	Leading coefficient	*y*-intercept of its graph
	$f(x) = 3x^2 - 2x + 1$	2	even	quadratic	3	1
a)	$g(x) = 5x + x^5 - 2x^3$	**5**	**odd**	**quintic**	**1**	**0**
b)	$h(x) = 2x^2 - 3x^3 - 7$	**3**	**odd**	**cubic**	**−3**	**−7**
c)	$k(x) = 5 - x^4 - 3x$	**4**	**even**	**quartic**	**−1**	**5**

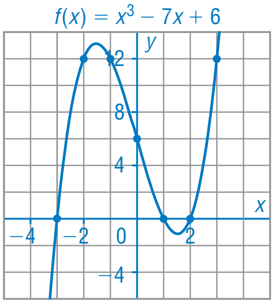
B

6. Use a table of values to sketch the graph of each polynomial function.

a) $f(x) = x^3 - 7x + 6$

The equation represents an odd-degree polynomial function. The leading coefficient is positive, so as $x \to -\infty$, the graph falls and as $x \to \infty$, the graph rises. The constant term is 6, so the y-intercept is 6.

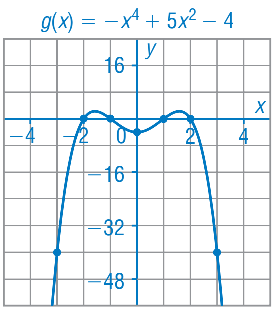
x	$f(x)$
−3	0
−2	12
−1	12
0	6
1	0
2	0
3	12



b) $g(x) = -x^4 + 5x^2 - 4$

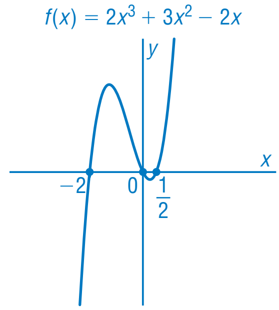
The equation represents an even-degree polynomial function. The leading coefficient is negative, so the graph opens down. The constant term is −4, so the y-intercept is −4.

x	$g(x)$
−3	−40
−2	0
−1	0
0	−4
1	0
2	0
3	−40



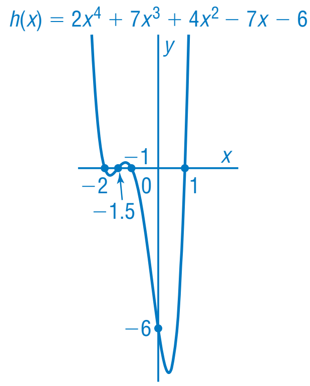
7. Use intercepts to sketch the graph of each polynomial function.

a) $f(x) = 2x^3 + 3x^2 - 2x$



Factor.
$f(x) = x(2x^2 + 3x - 2)$
$f(x) = x(x + 2)(2x - 1)$
Determine the zeros of $f(x)$. Let $f(x) = 0$.
$0 = x(x + 2)(2x - 1)$
The zeros are: $0, -2, \frac{1}{2}$
So, the x-intercepts of the graph are: $0, -2, \frac{1}{2}$
The equation has degree 3, so it is an odd-degree polynomial function.
The leading coefficient is positive, so as $x \to -\infty$, the graph falls and as $x \to \infty$, the graph rises.
The constant term is 0, so the y-intercept is 0.

b) $h(x) = 2x^4 + 7x^3 + 4x^2 - 7x - 6$



Factor the polynomial. Use the factor theorem.
The factors of the constant term, −6, are: 1, −1, 2, −2, 3, −3, 6, −6
Use mental math to substitute $x = 1$, then $x = -1$ in $h(x)$ to determine that both $x - 1$ and $x + 1$ are factors.
Divide by $x - 1$.

$$\begin{array}{r|rrrrr} 1 & 2 & 7 & 4 & -7 & -6 \\ & & 2 & 9 & 13 & 6 \\ \hline & 2 & 9 & 13 & 6 & 0 \end{array}$$

So, $2x^4 + 7x^3 + 4x^2 - 7x - 6 = (x - 1)(2x^3 + 9x^2 + 13x + 6)$
Divide $2x^3 + 9x^2 + 13x + 6$ by $x + 1$.

$$\begin{array}{r|rrrr} -1 & 2 & 9 & 13 & 6 \\ & & -2 & -7 & -6 \\ \hline & 2 & 7 & 6 & 0 \end{array}$$

So, $2x^4 + 7x^3 + 4x^2 - 7x - 6 = (x - 1)(x + 1)(2x^2 + 7x + 6)$
Factor the trinomial: $2x^2 + 7x + 6 = (2x + 3)(x + 2)$
So, $2x^4 + 7x^3 + 4x^2 - 7x - 6 = (x - 1)(x + 1)(2x + 3)(x + 2)$
Determine the zeros of $h(x)$. Let $h(x) = 0$.
$0 = (x - 1)(x + 1)(2x + 3)(x + 2)$
The zeros are: 1, −1, −1.5, −2
So, the x-intercepts of the graph are: 1, −1, −1.5, −2
The equation has degree 4, so it is an even-degree polynomial function.
The leading coefficient is positive, so the graph opens up. The constant term is −6, so the y-intercept is −6.

8. Identify the graph that corresponds to each function. Justify your choices.

a) $f(x) = -x^3 + 3x^2 + x - 3$

Odd degree, negative leading coefficient: graph B

b) $g(x) = x^4 - 3x^2 - 3$

Even degree, positive leading coefficient: graph D

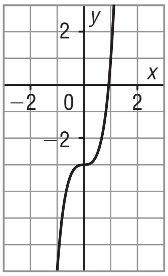
c) $h(x) = x^5 + 3x^3 - 3$

Odd degree, positive leading coefficient: graph A

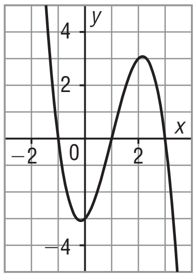
d) $k(x) = -x^2 + 4x - 3$

Even degree, negative leading coefficient: graph C

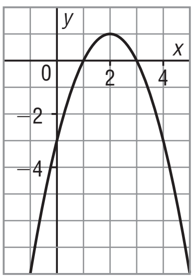
i) Graph A



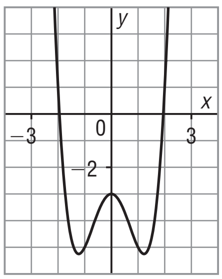
ii) Graph B



iii) Graph C



iv) Graph D



9. Determine the zeros of each polynomial function. State the multiplicity of each zero. How does the graph of each function behave at the related x-intercepts?
Use graphing technology to check.

a) $f(x) = (x + 3)^3$

$0 = (x + 3)^3$
Root of the equation: $x = -3$
Zero of the function: -3
The zero has multiplicity 3.
So, the graph crosses the x-axis at $x = -3$.

b) $g(x) = (x - 2)^2(x + 3)^2$

$0 = (x - 2)^2(x + 3)^2$
Roots of the equation: $x = 2$ and $x = -3$
Zeros of the function: 2 and -3
The zero 2 has multiplicity 2.
The zero -3 has multiplicity 2.
So, the graph just touches the x-axis at $x = 2$ and at $x = -3$.

c) $h(x) = (x - 1)^4(2x + 1)$

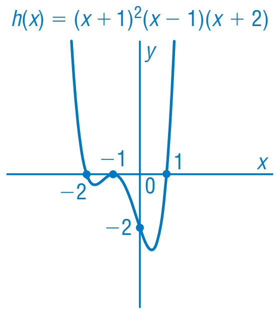
$0 = (x - 1)^4(2x + 1)$
Roots of the equation: $x = 1$ and $x = -0.5$
Zeros of the function: 1 and -0.5
The zero 1 has multiplicity 4.
The zero -0.5 has multiplicity 1.
So, the graph just touches the x-axis at $x = 1$ and crosses the x-axis at $x = -0.5$.

d) $j(x) = (x - 4)^3(x + 1)^2$

$0 = (x - 4)^3(x + 1)^2$
Roots of the equation: $x = 4$ and $x = -1$
Zeros of the function: 4 and -1
The zero 4 has multiplicity 3.
The zero -1 has multiplicity 2.
So, the graph crosses the x-axis at $x = 4$ and just touches the x-axis at $x = -1$.

10. Sketch the graph of this polynomial function.
$h(x) = (x + 1)^2(x - 1)(x + 2)$

To determine the roots, let $h(x) = 0$.
$0 = (x + 1)^2(x - 1)(x + 2)$
Zeros of the function: -1, 1, and -2
The zero -1 has multiplicity 2.
The zeros 1 and -2 have multiplicity 1.
So, the graph just touches the x-axis at $x = -1$ and crosses the x-axis at $x = 1$ and at $x = -2$.
The equation has degree 4, so it is an even-degree polynomial function.
The leading coefficient is positive, so the graph opens up. The y-intercept is: $(1)^2(-1)(2) = -2$



11. a) Write an equation in standard form for each polynomial function described below.

i) a cubic function with zeros 3, -3, and 0

Sample response:
The zeros of the function are the roots of its equation.
$y = x(x - 3)(x + 3)$
$y = x(x^2 - 9)$
$y = x^3 - 9x$

ii) a quartic function with zeros -2 and 1 of multiplicity 1, and a zero 2 of multiplicity 2

Sample response:
The zeros of the function are the roots of its equation.
$y = (x + 2)(x - 1)(x - 2)^2$
$y = (x^2 + x - 2)(x^2 - 4x + 4)$
$y = x^4 - 4x^3 + 4x^2 + x^3 - 4x^2 + 4x - 2x^2 + 8x - 8$
$y = x^4 - 3x^3 - 2x^2 + 12x - 8$

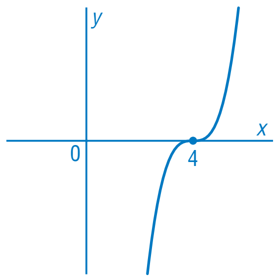
b) Is there more than one possible equation for each function in part a? Explain.

Yes, if I multiply the polynomial by a constant factor, I don't change the zeros but I do change the equation.

12. Sketch a possible graph of each polynomial function.

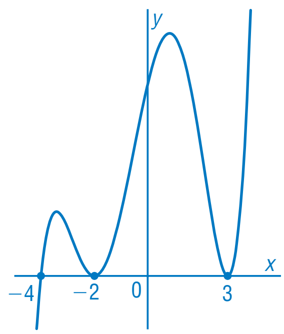
a) cubic function; leading coefficient is positive; zero of 4 has multiplicity 3

The zero has multiplicity 3, so the graph crosses the x-axis at $x = 4$. Since the function is cubic, there are no more zeros. The leading coefficient is positive so as $x \to -\infty$, the graph falls and as $x \to \infty$, the graph rises.



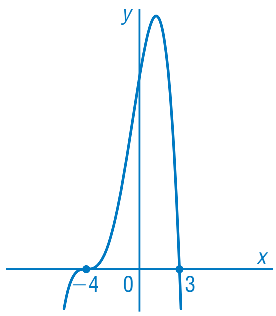
b) quintic function; leading coefficient is positive; zero of 3 has multiplicity 2; zero of -2 has multiplicity 2; zero of -4 has multiplicity 1

Each of the zeros 3 and -2 has multiplicity 2, so the graph just touches the x-axis at $x = 3$ and $x = -2$. The zero -4 has multiplicity 1, so the graph crosses the x-axis at $x = -4$. Since the function is quintic, there are no more zeros. The leading coefficient is positive, so as $x \to -\infty$, the graph falls and as $x \to \infty$, the graph rises.



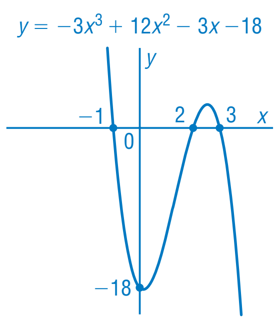
c) quartic function; leading coefficient is negative; zero of -4 has multiplicity 3; zero of 3 has multiplicity 1

The zero -4 has multiplicity 3, so the graph crosses the x-axis at $x = -4$. The zero 3 has multiplicity 1, so the graph crosses the x-axis at $x = 3$. Since the function is quartic, there are no more zeros. The leading coefficient is negative, so the graph opens down.



13. A cubic function has zeros 2, 3, and -1. The y-intercept of its graph is -18. Sketch the graph, then determine an equation of the function.

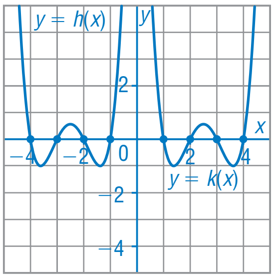
The zeros of the function are the roots of its equation.
Let k represent the leading coefficient.
$y = k(x - 2)(x - 3)(x + 1)$
The constant term in the equation is -18.
So, $k(-2)(-3)(1) = -18$
$k = -3$
So, an equation is:
$y = -3(x - 2)(x - 3)(x + 1)$
$y = -3(x^2 - 5x + 6)(x + 1)$
$y = -3x^3 + 12x^2 - 3x - 18$



C

14. Investigate pairs of graphs of even-degree polynomial functions of the form shown below for different values of the variables a, b, c, and $d \in \mathbb{Z}$. Describe one graph as a transformation image of the other graph. What conclusions can you make?
$h(x) = (x + a)(x + b)(x + c)(x + d)$ and
$k(x) = (x - a)(x - b)(x - c)(x - d)$

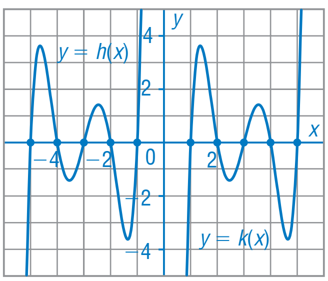
Sample response:



The graph of $k(x) = (x - 1)(x - 2)(x - 3)(x - 4)$ is the image of the graph of $h(x) = (x + 1)(x + 2)(x + 3)(x + 4)$ after a reflection in the y-axis. In general, the graph of $k(x)$ is the image of the graph of $h(x)$ after a reflection in the y-axis.

15. Investigate pairs of graphs of odd-degree polynomial functions of the form shown below for different values of the variables a, b, c, d, and $e \in \mathbb{Z}$. Describe one graph as a transformation image of the other graph. What conclusions can you make?
$h(x) = (x + a)(x + b)(x + c)(x + d)(x + e)$ and
$k(x) = (x - a)(x - b)(x - c)(x - d)(x - e)$

Sample response:

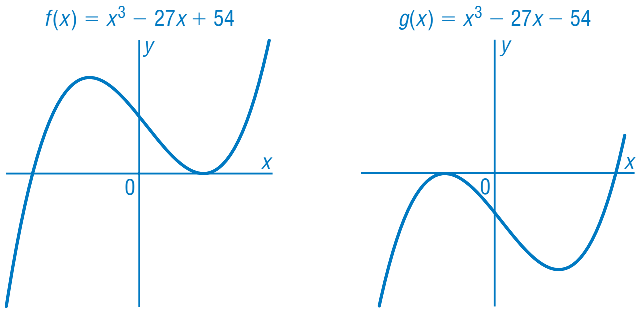


The graph of $k(x) = (x - 1)(x - 2)(x - 3)(x - 4)(x - 5)$ is the image of the graph of $h(x) = (x + 1)(x + 2)(x + 3)(x + 4)(x + 5)$ after a rotation of 180° about the origin. In general, the graph of $k(x)$ is the image of the graph of $h(x)$ after a rotation of 180° about the origin.

16. Each of the functions $f(x) = x^3 - 27x + 54$ and $g(x) = x^3 - 27x - 54$ has one zero of multiplicity 2 and one different zero. Use only this information to determine the values of b for which the function $h(x) = x^3 - 27x + b$ has each number of zeros. Explain your strategy.

a) 3 different zeros

I sketched the graphs.



Each of the graphs of $f(x)$ and $g(x)$ just touches the x-axis at the point that corresponds to the zero of multiplicity 2. The graph of $f(x)$ has y-intercept 54 and the graph of $g(x)$ has y-intercept -54. So, for the graph of $h(x)$ to have 3 different zeros, the graph of $h(x)$ must lie between the graphs of $f(x)$ and $g(x)$. So, $-54 < b < 54$

b) 1 zero of multiplicity 1 and no other zeros

For the graph of $h(x)$ to have 1 zero of multiplicity 1 and no other zeros, the graph of $f(x)$ must be translated up or the graph of $g(x)$ must be translated down so that the local minimum point lies above the x-axis or the local maximum point lies below the x-axis. So, $b > 54$ or $b < -54$