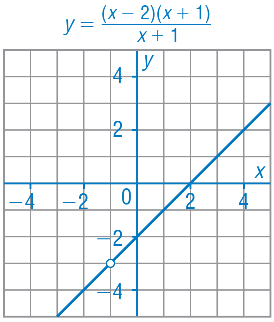
Lesson 2.4 Exercises, pages 134–140

A

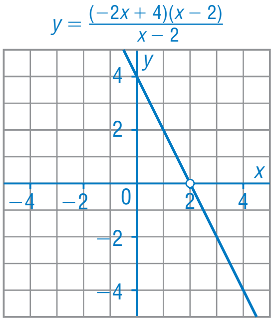
3. Sketch the graph of each function.

a) $y = \dfrac{(x - 2)(x + 1)}{x + 1}$



The function is undefined when: $x = -1$
There is a hole at $x = -1$.
The function can be written as: $y = x - 2, x \neq -1$
The y-coordinate of the hole is: $y = -3$
Draw an open circle at $(-1, -3)$.
When $x = 0, y = -2$
When $y = 0, x = 2$
Draw the line $y = x - 2$ on either side of the hole.

b) $y = \dfrac{(-2x + 4)(x - 2)}{x - 2}$

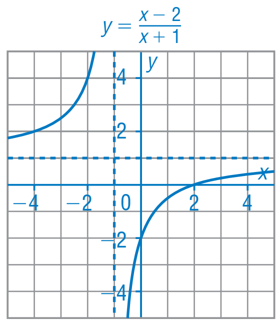


The function is undefined when: $x = 2$
There is a hole at $x = 2$.
The function can be written as: $y = -2x + 4, x \neq 2$
The y-coordinate of the hole is: $y = 0$
Draw an open circle at $(2, 0)$.
When $x = 0, y = 4$
Draw the line $y = -2x + 4$ on either side of the hole.

4. Sketch the graph of each function.

a) $y = \dfrac{x - 2}{x + 1}$ **b)** $y = \dfrac{-2x + 4}{x}$

a)

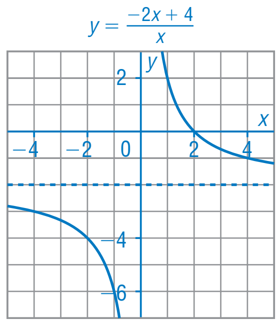


The function is undefined when:
$x = -1$
There are no common factors, so there are no holes. The vertical asymptote has equation: $x = -1$
There is a horizontal asymptote. The numerator and denominator have equal leading coefficients, so the horizontal asymptote has equation $y = 1$.
Close to the asymptotes:

x	-1.01	-0.99	-100	100
y	301	-299	1.03	0.97

Some of the y-values above are approximate.
When $x = 0$, $y = -2$
When $y = 0$, $x = 2$
Determine the coordinates of some other points:
$(-2, 4)$, $(-4, 2)$
Draw broken lines for the asymptotes. Join the points to form smooth curves.

b)



The function is undefined when:
$x = 0$
There are no common factors, so there are no holes.
The vertical asymptote has equation: $x = 0$
There is a horizontal asymptote.
The leading coefficients are -2 and 1, so the horizontal asymptote has equation $y = -2$.
Close to the asymptotes:

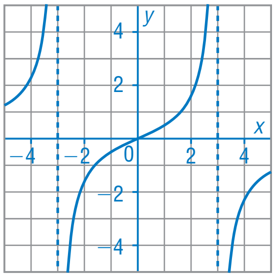
x	-0.01	0.01	-100	100
y	-402	398	-2.04	-1.96

When $y = 0$, $x = 2$
Determine the coordinates of some other points: $(-4, -3)$, $(-1, -6)$, $(1, 2)$, $(4, -1)$
Draw broken lines for the asymptotes. Join the points to form smooth curves.

B

5. Sketch the graph of each function, then state the domain.

a) $y = \frac{-4x}{x^2 - 9}$



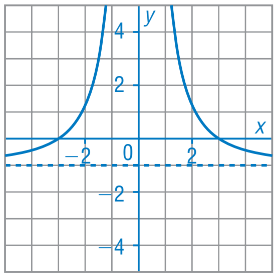
The function is undefined when: $x = \pm 3$
There are no common factors, so there are no holes.
The vertical asymptotes have equations: $x = -3$ and $x = 3$
There is a horizontal asymptote with equation $y = 0$.
Close to the asymptotes:

x	-3.01	-2.99	2.99
y	200	-200	200

x	3.01	-100	100
y	-200	0.04	-0.04

Some of the y-values above are approximate.
When $x = 0$, $y = 0$
Determine the approximate coordinates of some other points: $(-4, 2.3)$, $(-2, -1.6)$, $(2, 1.6)$, $(4, -2.3)$
Draw broken lines for the asymptotes. Join the points to form smooth curves.
The domain is: $x \neq \pm 3$

b) $y = \frac{x^2 - 9}{-x^2}$

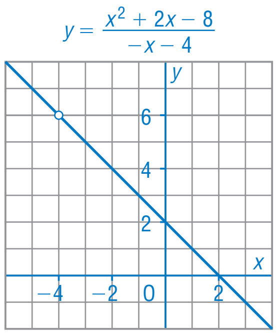


The function is undefined when: $x = 0$
There are no common factors, so there are no holes. The vertical asymptote has equation: $x = 0$
There is a horizontal asymptote.
The leading coefficients are 1 and -1, so the equation of the horizontal asymptote is $y = -1$.
Close to the asymptotes:

x	± 0.1	± 100
y	899	-0.9991

When $y = 0$, $x = \pm 3$
Determine the coordinates of some other points: $(\pm 2, 1.25)$
Draw broken lines for the asymptotes. Join the points to form smooth curves.
The domain is: $x \neq 0$

c) $y = \dfrac{x^2 + 2x - 8}{-x - 4}$

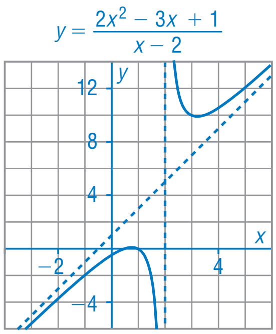


The function is undefined when:
$x = -4$

Factor: $y = \dfrac{(x + 4)(x - 2)}{-(x + 4)}$

There is a hole at $x = -4$.
The function can be written as:
$y = -x + 2, x \neq -4$
The y-coordinate of the hole is:
$y = 6$
Draw an open circle at $(-4, 6)$, then draw the line $y = -x + 2$ on either side of the hole.
The domain is: $x \neq -4$

d) $y = \dfrac{2x^2 - 3x + 1}{x - 2}$



The function is undefined when:
$x = 2$
There are no common factors, so there are no holes.
The vertical asymptote has equation: $x = 2$
There is also an oblique asymptote.
Determine:
$(2x^2 - 3x + 1) \div (x - 2)$

2	2	−3	1
		4	2
	2	1	3

The quotient is $2x + 1$; so the equation of the oblique asymptote is $y = 2x + 1$.
Draw broken lines for the asymptotes.
Close to the vertical asymptote:

x	1.99	2.01
y	$\doteq -295$	$\doteq 305$

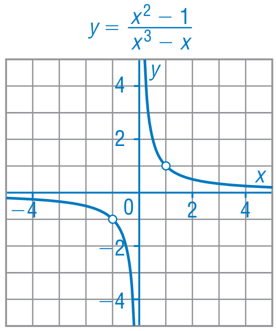
When $x = 0$, $y = -0.5$
When $y = 0$, $2x^2 - 3x + 1 = 0$
$(2x - 1)(x - 1) = 0$
$x = 0.5$ or $x = 1$
Plot points at $(0, -0.5)$, $(0.5, 0)$, and $(1, 0)$.
Determine the coordinates of some other points: $(-1, -2)$, $(3, 10)$, $(4, 10.5)$
Join the points to form smooth curves.
The domain is: $x \neq 2$

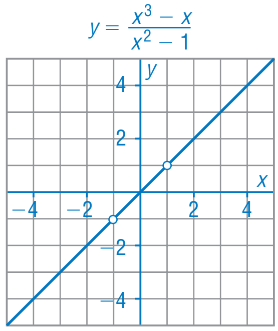
6. a) How are these functions different from other functions in this lesson?

i) $y = \frac{x^2 - 1}{x^3 - x}$ **ii)** $y = \frac{x^3 - x}{x^2 - 1}$

Both functions contain x^3-terms.

b) Sketch the graph of each function in part a, then state the domain and range.





i) Factor: $y = \frac{(x - 1)(x + 1)}{x(x - 1)(x + 1)}$

There are holes at $x = \pm 1$, and an asymptote with equation: $x = 0$

The function can be written as: $y = \frac{1}{x}$, $x \neq \pm 1$

The coordinates of the holes are: (−1, −1) and (1, 1)

Draw open circles at the holes.

Graph $y = \frac{1}{x}$ on either side of each hole.

The domain is: $x \neq \pm 1, x \neq 0$

The range is: $y \neq \pm 1, y \neq 0$

ii) Factor: $y = \frac{x(x - 1)(x + 1)}{(x - 1)(x + 1)}$

There are holes at $x = \pm 1$.

The function can be written as: $y = x, x \neq \pm 1$

The coordinates of the holes are: (−1, −1) and (1, 1)

Draw open circles at the holes.

Graph $y = x$ on either side of the holes.

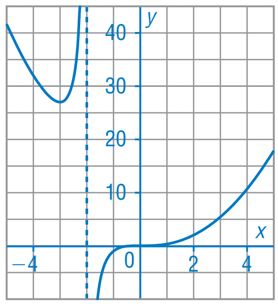
The domain is: $x \neq \pm 1$

The range is: $y \neq \pm 1$

7. For a rational function, when the degree of the numerator is 2 or more than the degree of the denominator, the graph has no horizontal or oblique asymptotes. Without using graphing technology, determine a strategy to sketch the graph of $y = \frac{x^3}{x + 2}$ then graph the function. State the domain.

The function is undefined when $x = -2$.
Draw a vertical asymptote at $x = -2$.
Make a table of values.
Approximate the y-values.

$y = \frac{x^3}{x + 2}$



x	−5	−4	−3	−2.01	−1.99	−1	0	1	2	3	4
y	42	32	27	812	−788	−1	0	0.3	2	5.4	11

Join the points with 2 smooth curves.
The domain is: $x \neq -2$

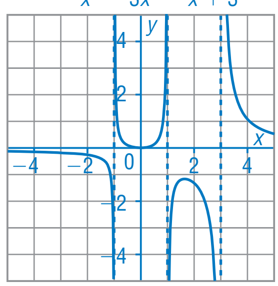
C

8. Sketch the graph of each function, then state the domain.

a) $y = \frac{x^2}{x^3 - 3x^2 - x + 3}$

The function is undefined when:
$x^3 - 3x^2 - x + 3 = 0$
Use the factor theorem.
Let $f(x) = x^3 - 3x^2 - x + 3$
Use mental math to determine $f(1) = 0$ and $f(-1) = 0$.
$f(3) = 3^3 - 3(3)^2 - 3 + 3$
$= 0$
So, there are vertical asymptotes with equations:
$x = -1$, $x = 1$, and $x = 3$
Since the degree of the numerator is less than the degree of the denominator, there is a horizontal asymptote with equation $y = 0$.
Close to the asymptotes:

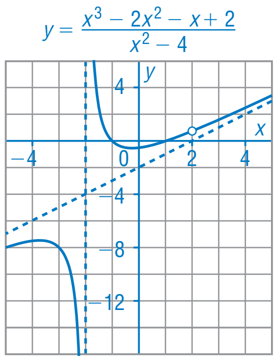
$y = \frac{x^2}{x^3 - 3x^2 - x + 3}$



x	−100	−1.01	−0.99	0.99	1.01	2	2.99	3.01	100
y	−0.01	−13	12	25	−26	$-1.\overline{3}$	−113	112	0.01

When $x = 0$, $y = 0$
Determine the approximate coordinates of other points: $(-2, -0.3)$, $(\pm 0.5, 0.1)$, $(4, 1.1)$
Draw 4 smooth curves through the points.
The domain is: $x \neq -1$, $x \neq 1$, $x \neq 3$

b) $y = \dfrac{x^3 - 2x^2 - x + 2}{x^2 - 4}$



The function is undefined when:

$x^2 - 4 = 0$

$x = \pm 2$

Factor the numerator. Use the factor theorem.

Let $f(x) = x^3 - 2x^2 - x + 2$

Use mental math to determine $f(1) = 0$ and $f(-1) = 0$.

$f(2) = 2^3 - 2(2)^2 - 2 + 2$

$= 0$

The function is: $y = \dfrac{(x - 1)(x + 1)(x - 2)}{(x - 2)(x + 2)}$

There is a hole at $x = 2$. The function can be written as:

$y = \dfrac{(x - 1)(x + 1)}{(x + 2)}, x \neq 2$, or $y = \dfrac{x^2 - 1}{x + 2}, x \neq 2$

There is a vertical asymptote at $x = -2$.

The y-coordinate of the hole is 0.75.

Since the degree of the numerator is 1 more than the degree of the denominator, there is an oblique asymptote. Determine:

$(x^2 - 1) \div (x + 2)$

−2	1	0	−1
		−2	4
	1	−2	3

The quotient is $x - 2$; so the equation of the oblique asymptote is $y = x - 2$.

Draw broken lines for the asymptotes.

When $x = 0$, $y = -0.5$

When $y = 0$, $x^2 - 1 = 0$, and $x = \pm 1$

Choose points close to the asymptotes and other points:

x	−4	−3	−2.01	−1.99
y	−7.5	−8	−304	296

Draw an open circle at (2, 0.75).

Join the points to form 2 smooth curves.

The domain is: $x \neq \pm 2$