CUMULATIVE REVIEW Chapters 1-2, pages 157–160

1

1. Use long division to divide. Write the division statement.
$(-x^2 - 5x^3 + 2x - 3x^4 + 3) \div (x - 2)$

Write the polynomial in descending order:
$-3x^4 - 5x^3 - x^2 + 2x + 3$

$$
\begin{array}{r}
-3x^3 - 11x^2 - 23x - 44 \\
x - 2 \overline{)\,-3x^4 - 5x^3 - x^2 + 2x + 3} \\
\underline{-3x^4 + 6x^3} \\
-11x^3 - x^2 \\
\underline{-11x^3 + 22x^2} \\
-23x^2 + 2x \\
\underline{-23x^2 + 46x} \\
-44x + 3 \\
\underline{-44x + 88} \\
-85
\end{array}
$$

$-3x^4 - 5x^3 - x^2 + 2x + 3 = (x - 2)(-3x^3 - 11x^2 - 23x - 44) - 85$

2. Use synthetic division to divide. Write the division statement.
$(7x + x^4 - 8x^3) \div (x - 5)$

Write the polynomial in descending order:
$x^4 - 8x^3 + 7x$
Use zeros as placeholders.

$$
\begin{array}{r|rrrrr}
5 & 1 & -8 & 0 & 7 & 0 \\
 & & 5 & -15 & -75 & -340 \\
\hline
 & 1 & -3 & -15 & -68 & -340
\end{array}
$$

$x^4 - 8x^3 + 7x = (x - 5)(x^3 - 3x^2 - 15x - 68) - 340$

3. Determine the remainder: $(4x^4 - 7x^3 + 2x^2 + x) \div (x + 1)$

$x + 1 = x - (-1)$
Let $P(x) = 4x^4 - 7x^3 + 2x^2 + x$
$P(-1) = 4(-1)^4 - 7(-1)^3 + 2(-1)^2 + (-1)$
$= 4 + 7 + 2 - 1$
$= 12$
The remainder is 12.

4. Fully factor this polynomial: $6x^3 - 11x^2 - 3x + 2$

Let $P(x) = 6x^3 - 11x^2 - 3x + 2$
The factors of 2 are: 1, −1, 2, −2
Use mental math to substitute $x = 1$, then $x = -1$
to determine that neither $x - 1$ nor $x + 1$ is a factor.
Try $x = 2$: $P(2) = 6(2)^3 - 11(2)^2 - 3(2) + 2$
$= 0$
So, $x - 2$ is a factor.
Divide to determine the other factor.

2	6	−11	−3	2
		12	2	−2
	6	1	−1	0

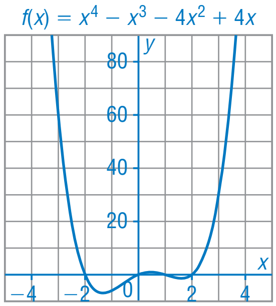
So, $6x^3 - 11x^2 - 3x + 2 = (x - 2)(6x^2 + x - 1)$
Factor the trinomial: $6x^2 + x - 1 = (3x - 1)(2x + 1)$
So, $6x^3 - 11x^2 - 3x + 2 = (x - 2)(3x - 1)(2x + 1)$

5. Sketch a graph of each polynomial function.

a) $f(x) = x^4 - x^3 - 4x^2 + 4x$

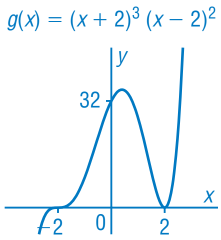
The equation represents an even-degree polynomial function.
The leading coefficient is positive, so the graph opens up.
The constant term is 0, so the y-intercept is 0.
Make a table of values. Plot the points then join them with a smooth curve.

x	$f(x)$
−3	60
−2	0
−1	−6
0	0
1	0
2	0
3	30



b) $g(x) = (x + 2)^3(x - 2)^2$

To determine the roots, let $g(x) = 0$.
$0 = (x + 2)^3(x - 2)^2$
Zeros of the function are: -2 and 2
The zero -2 has multiplicity 3.
The zero 2 has multiplicity 2.
So, the graph crosses the x-axis at $x = -2$
and just touches the x-axis at $x = 2$.
The equation has degree 5, so it is an
odd-degree polynomial function.
The leading coefficient is positive, so as $x \to -\infty$, the graph falls and
as $x \to \infty$, the graph rises.
The y-intercept is: $(2)^3(-2)^2 = 32$
Plot points at the intercepts then join them with a smooth curve.



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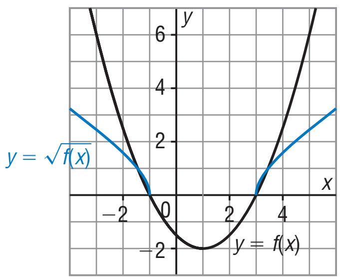
6. For the graph of the quadratic function $y = f(x)$ below:

a) Sketch the graph of $y = \sqrt{f(x)}$.

Mark points where $y = 0$ or $y = 1$.
Choose, then mark other points
on the graph of $y = \sqrt{f(x)}$.

x	$y = f(x)$	$y = \sqrt{f(x)}$
-3	6	$\sqrt{6} \doteq 2.4$
5	6	$\sqrt{6} \doteq 2.4$

Join all the points with 2 smooth curves.



b) State the domain and range of $y = \sqrt{f(x)}$.

Domain is: $x \le -1, x \ge 3$
Range is: $y \ge 0$

7. Use graphing technology to solve each equation. Give the solution to the nearest tenth.

a) $\sqrt{4 - x} + 3 = 2 + \sqrt{2x}$

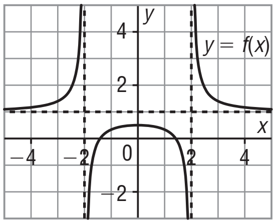
Graph the related function:
$f(x) = \sqrt{4 - x} + 1 - \sqrt{2x}$
The approximate zero is:
2.4867591
So, the root is: $x \doteq 2.5$

b) $\dfrac{-3}{x^2 - 2} = \dfrac{5}{x - 2}$

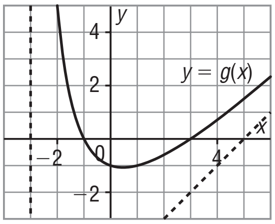
Graph the related function:
$f(x) = \dfrac{-3}{x^2 - 2} - \dfrac{5}{x - 2}$
The approximate zeros are:
-2.113836 and 1.5138357
So, the roots are: $x \doteq -2.1$ and
$x \doteq 1.5$

8. Match each function below to its graph. Justify your choice.

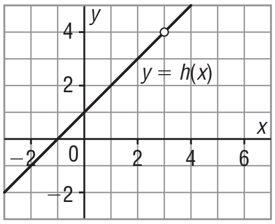
i) Graph A



ii) Graph B



iii) Graph C



a) $y = \dfrac{x^2 - 2x - 3}{x + 3}$

The numerator can be factored.

$y = \dfrac{(x - 3)(x + 1)}{x + 3}$

There are no common factors. Since $x + 3 \neq 0$, $x = -3$ is a vertical asymptote. The degree of the numerator is 1 more than that of the denominator, so there is an oblique asymptote. The function matches Graph B.

b) $y = \dfrac{x^2 - 2}{x^2 - 4}$

Since $x^2 - 4 \neq 0$, $x \neq \pm 2$, so $x = 2$ and $x = -2$ are vertical asymptotes. The degrees of the numerator and denominator are the same so there is a horizontal asymptote. As $|x| \rightarrow \infty$,

$y = \dfrac{x^2 - 2}{x^2 - 4} \rightarrow y = \dfrac{x^2}{x^2}$, or $y = 1$,

which is the horizontal asymptote. The function matches Graph A.

c) $y = \dfrac{x^2 - 2x - 3}{x - 3}$

The numerator can be factored.

$y = \dfrac{(x - 3)(x + 1)}{x - 3}$

Since the numerator and denominator have a common factor, there is a hole where the denominator $x - 3 = 0$; that is, at $x = 3$. The function matches Graph C.