Lesson 3.3 Exercises, pages 201–210

A

3. Here is the graph of $y = g(x)$. On the same grid, sketch the graph of each function.

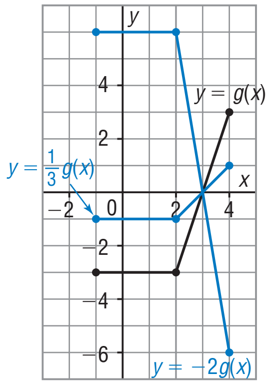
a) $y = \frac{1}{3}g(x)$

$a = \frac{1}{3}$, so the graph of $y = g(x)$ is vertically compressed by a factor of $\frac{1}{3}$.

Use: (x, y) on $y = g(x)$ corresponds to $\left(x, \frac{1}{3}y\right)$ on $y = \frac{1}{3}g(x)$.

Point on $y = g(x)$	Point on $y = \frac{1}{3}g(x)$
(−1, −3)	(−1, −1)
(2, −3)	(2, −1)
(4, 3)	(4, 1)

Plot the points, then join them.



b) $y = -2g(x)$

$a = -2$, so the graph of $y = g(x)$ is vertically stretched by a factor of 2, then reflected in the x-axis.
Use: (x, y) on $y = g(x)$ corresponds to $(x, -2y)$ on $y = -2g(x)$.

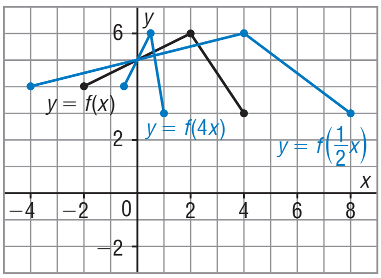
Point on $y = g(x)$	Point on $y = -2g(x)$
(−1, −3)	(−1, 6)
(2, −3)	(2, 6)
(4, 3)	(4, −6)

Plot the points, then join them.

4. Here is the graph of $y = f(x)$. On the same grid, sketch the graph of each function.

a) $y = f(4x)$

$b = 4$, so the graph of $y = f(x)$ is horizontally compressed by a factor of $\frac{1}{4}$.
Use: (x, y) on $y = f(x)$ corresponds to $\left(\frac{x}{4}, y\right)$ on $y = f(4x)$.



Point on $y = f(x)$	Point on $y = f(4x)$
(−2, 4)	(−0.5, 4)
(2, 6)	(0.5, 6)
(4, 3)	(1, 3)

Plot the points, then join them.

b) $y = f\left(\frac{1}{2}x\right)$

$b = \frac{1}{2}$ or 0.5, so the graph of $y = f(x)$ is horizontally stretched by a factor of $\frac{1}{0.5}$, or 2.

Use: (x, y) on $y = f(x)$ corresponds to $\left(\frac{x}{0.5}, y\right)$, or $(2x, y)$ on $y = f\left(\frac{1}{2}x\right)$.

Point on $y = f(x)$	Point on $y = f\left(\frac{1}{2}x\right)$
(−2, 4)	(−4, 4)
(2, 6)	(4, 6)
(4, 3)	(8, 3)

Plot the points, then join them.

5. The graph of $y = f(x)$ is transformed as described below. Write an equation of the image graph in terms of the function f.

a) a vertical stretch by a factor of 4

The equation of the image graph has the form $y = af(x)$.
Since the graph was vertically stretched by a factor of 4, $a = 4$.
So, the equation of the image graph is: $y = 4f(x)$

b) a horizontal compression by a factor of $\frac{1}{3}$ and a reflection in the y-axis

The equation of the image graph has the form $y = f(bx)$.
Since the graph was horizontally compressed by a factor of $\frac{1}{3}$, $\frac{1}{b} = \frac{1}{3}$, or $b = 3$. Since the graph was also reflected in the y-axis, b is negative. So, the equation of the image graph is: $y = f(-3x)$

c) a vertical compression by a factor of $\frac{1}{5}$ and a reflection in the x-axis

The equation of the image graph has the form $y = af(x)$.
Since the graph was vertically compressed by a factor of $\frac{1}{5}$, $a = \frac{1}{5}$
Since the graph was also reflected in the x-axis, a is negative.
So, the equation of the image graph is: $y = -\frac{1}{5}f(x)$

B

6. The graph of $y = |x|$ is transformed, and the equation of its image is $y = |2x|$. Student A says the graph of $y = |x|$ was horizontally compressed by a factor of $\frac{1}{2}$. Student B says the graph of $y = |x|$ was vertically stretched by a factor of 2. Who is correct? Explain.

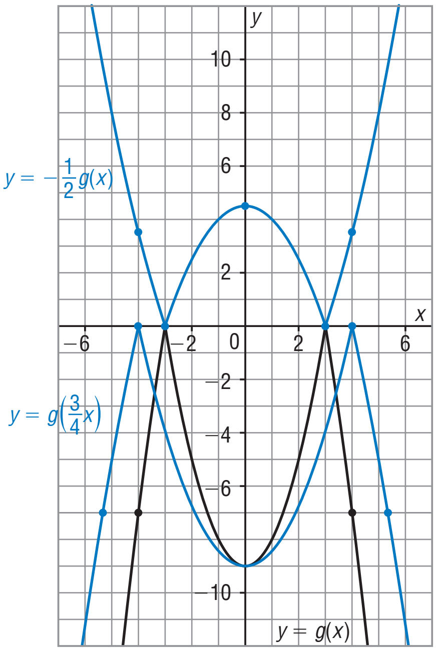
Both students are correct. Compare $y = |2x|$ to $y = a|bx|$: $a = 1$ and $b = 2$.
So, the graph of $y = |x|$ was horizontally compressed by a factor of $\frac{1}{2}$.
Because $|2|$ is 2, the equation $y = |2x|$ can also be written as $y = 2|x|$.
Compare $y = 2|x|$ to $y = a|bx|$: $a = 2$ and $b = 1$. So, the graph of $y = |x|$ was vertically stretched by a factor of 2.

7. The point A(36, 6) lies on the graph of $y = \sqrt{x}$. What are the coordinates of its image A′ on the graph of $y = -\frac{1}{2}\sqrt{3x}$? How do you know?

Compare $y = -\frac{1}{2}\sqrt{3x}$ to $y = a\sqrt{bx}$: $a = -\frac{1}{2}$ and $b = 3$
Point (x, y) on $y = \sqrt{x}$ corresponds to point $\left(\frac{x}{3}, -\frac{1}{2}y\right)$ on $y = -\frac{1}{2}\sqrt{3x}$.
So, the image of A(36, 6) is $A'\left(\frac{36}{3}, -\frac{1}{2}(6)\right)$, which is A′(12, −3).

8. Here is the graph of $y = g(x)$. On the same grid, sketch the graph of each function. State the domain and range of each function.

a) $y = g\left(\frac{3}{4}x\right)$



$b = \frac{3}{4}$, so the graph of $y = g(x)$ is horizontally stretched by a factor of $\frac{1}{0.75}$, or $\frac{4}{3}$.

Use: (x, y) on $y = g(x)$ corresponds to $\left(\frac{4}{3}x, y\right)$ on $y = g\left(\frac{3}{4}x\right)$.

Point on $y = g(x)$	Point on $y = g\left(\frac{3}{4}x\right)$
$(-4, -7)$	$\left(-\frac{16}{3}, -7\right)$
$(-3, 0)$	$(-4, 0)$
$(0, -9)$	$(0, -9)$
$(3, 0)$	$(4, 0)$
$(4, -7)$	$\left(\frac{16}{3}, -7\right)$

Plot the points, then join them with a smooth curve.

Both functions have domain: $x \in \mathbb{R}$

Both functions have range: $y \leq 0$

b) $y = -\frac{1}{2}g(x)$

$a = -\frac{1}{2}$, so the graph of $y = g(x)$ is vertically compressed by a factor of $\frac{1}{2}$, then reflected in the x-axis. Use: (x, y) on $y = g(x)$ corresponds to $\left(x, -\frac{1}{2}y\right)$ on $y = -\frac{1}{2}g(x)$.

Point on $y = g(x)$	Point on $y = -\frac{1}{2}g(x)$
$(-4, -7)$	$\left(-4, \frac{7}{2}\right)$
$(-3, 0)$	$(-3, 0)$
$(0, -9)$	$(0, 4.5)$
$(3, 0)$	$(3, 0)$
$(4, -7)$	$\left(4, \frac{7}{2}\right)$

Plot the points, then join them with a smooth curve.

Both functions have domain: $x \in \mathbb{R}$

The range of $y = g(x)$ is: $y \leq 0$

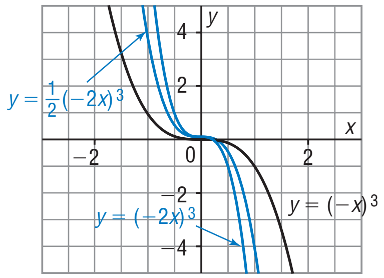
The range of $y = -\frac{1}{2}g(x)$ is: $y \geq 0$

9. On each grid, sketch the graph of each given function then state its domain and range.

a) i) $y = (-2x)^3$

Compare to $y = (-x)^3$: $b = 2$; so use mental math and the transformation: (x, y) on $y = (-x)^3$ corresponds to $\left(\frac{x}{2}, y\right)$ on $y = (-2x)^3$.

Domain: $x \in \mathbb{R}$; range: $y \in \mathbb{R}$



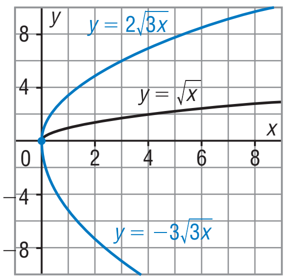
ii) $y = \frac{1}{2}(-2x)^3$

Compare to $y = (-2x)^3$: $a = \frac{1}{2}$; so use mental math and the transformation: (x, y) on $y = (-2x)^3$ corresponds to $\left(x, \frac{1}{2}y\right)$ on $y = \frac{1}{2}(-2x)^3$. Domain: $x \in \mathbb{R}$; range: $y \in \mathbb{R}$

b) i) $y = 2\sqrt{3x}$

Compare to $y = \sqrt{x}$: $a = 2$, $b = 3$; so use mental math and the transformation: (x, y) on $y = \sqrt{x}$ corresponds to $\left(\frac{x}{3}, 2y\right)$ on $y = 2\sqrt{3x}$.

Domain: $x \geq 0$; range: $y \geq 0$



ii) $y = -3\sqrt{3x}$

Compare to $y = \sqrt{x}$: $a = -3$, $b = 3$; so use mental math and the transformation: (x, y) on $y = \sqrt{x}$ corresponds to $\left(\frac{x}{3}, -3y\right)$ on $y = -3\sqrt{3x}$. Domain: $x \geq 0$; range: $y \leq 0$

10. The function $f(x) = (x - 10)(x + 8)$ has zeros at 10 and –8. What are the zeros of the function $y = 4f\left(\frac{1}{3}x\right)$?

Each point (x, y) on $y = f(x)$ corresponds to the point $\left(\frac{x}{\frac{1}{3}}, 4y\right)$, or $(3x, 4y)$ on $y = 4f\left(\frac{1}{3}x\right)$.

So, the zeros of $y = 4f\left(\frac{1}{3}x\right)$ are 3(10), or 30, and 3(−8), or −24.

11. Use transformations to describe how the graph of the second function compares to the graph of the first function.

a) $y = 3x + 4$ $\quad y = -\frac{1}{2}(3(5x) + 4)$

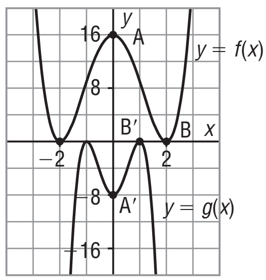
Let $f(x) = 3x + 4$, then compare $y = -\frac{1}{2}(3(5x) + 4)$ to $y = af(bx)$:
$a = -\frac{1}{2}$ and $b = 5$.
The graph of $y = -\frac{1}{2}(3(5x) + 4)$ is the image of the graph of $y = 3x + 4$ after a vertical compression by a factor of $\frac{1}{2}$, a horizontal compression by a factor of $\frac{1}{5}$, and a reflection in the x-axis.

b) $y = x^3 - 6x$ $\quad y = \frac{1}{4}\left[\left(-\frac{1}{2}x\right)^3 - 6\left(-\frac{1}{2}x\right)\right]$

Let $f(x) = x^3 - 6x$, then compare $y = \frac{1}{4}\left[\left(-\frac{1}{2}x\right)^3 - 6\left(-\frac{1}{2}x\right)\right]$ to $y = af(bx)$: $a = \frac{1}{4}$ and $b = -\frac{1}{2}$.

The graph of $y = \frac{1}{4}\left[\left(-\frac{1}{2}x\right)^3 - 6\left(-\frac{1}{2}x\right)\right]$ is the image of the graph of $y = x^3 - 6x$ after a vertical compression by a factor of $\frac{1}{4}$, a horizontal stretch by a factor of $\frac{1}{0.5}$, or 2, and a reflection in the y-axis

12. The graph of $y = g(x)$ is a transformation image of the graph of $y = f(x)$. Corresponding points are labelled. Write an equation of the image graph in terms of the function f.



Corresponding points are: A(0, 16) and A′(0, −8); B(2, 0) and B′(1, 0).
An equation for the image graph after a vertical or horizontal stretch can be written in the form $y = af(bx)$.
A point (x, y) on $y = f(x)$ corresponds to the point $\left(\frac{x}{b}, ay\right)$ on $y = af(bx)$.

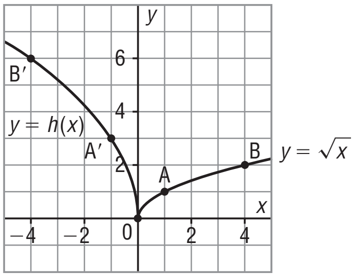
The image of A(0, 16) is $\left(\frac{0}{b}, a(16)\right)$, which is A′(0, −8).

Equate the y-coordinates: $a = -\frac{1}{2}$

The image of B(2, 0) is $\left(\frac{2}{b}, a(0)\right)$, which is B′(1, 0).

Equate the x-coordinates: $b = 2$
So, an equation of $y = g(x)$ is: $y = -\frac{1}{2}f(2x)$

13. The graph of $y = h(x)$ is a transformation image of the graph of $y = \sqrt{x}$. Corresponding points are labelled. Write an equation of the image graph in terms of x.



Corresponding points are: A(1, 1) and A′(−1, 3)
An equation for the image graph after a vertical or horizontal stretch can be written in the form $y = a\sqrt{bx}$.

A point (x, y) on $y = \sqrt{x}$ corresponds to the point $\left(\frac{x}{b}, ay\right)$ on $y = a\sqrt{bx}$.

So, the image of A(1, 1) is $\left(\frac{1}{b}, a(1)\right)$, which is A′(−1, 3).

Equate the x-coordinates:

$\frac{1}{b} = -1$

$b = -1$

Equate the y-coordinates:

$a = 3$

So, an equation is: $y = 3\sqrt{-x}$
Verify with a different pair of corresponding points.

B(4, 2) lies on $y = \sqrt{x}$ so $\left(\frac{4}{-1}, 3(2)\right)$, or (−4, 6) should lie on $y = h(x)$, which it does.
So, the equation $y = 3\sqrt{-x}$ is likely correct.

14. a) Determine the equation of the function $y = \sqrt{x}$ after each transformation.

i) a horizontal compression by a factor of $\frac{1}{9}$

The graph of $y = \sqrt{bx}$ is the image of the graph of $y = \sqrt{x}$ after a horizontal compression by a factor of $\frac{1}{b}$. Since the graph of $y = \sqrt{x}$ was horizontally compressed by a factor of $\frac{1}{9}$, $b = 9$ and the equation of the image graph is $y = \sqrt{9x}$, or $y = 3\sqrt{x}$.

ii) a vertical stretch by a factor of 3

The graph of $y = a\sqrt{x}$ is the image of the graph of $y = \sqrt{x}$ after a vertical stretch by a factor of a. Since the graph of $y = \sqrt{x}$ was vertically stretched by a factor of 3, $a = 3$ and the equation of the image graph is $y = 3\sqrt{x}$.

b) What do you notice about the equations in part a? Explain.

The equations in part a are the same. When writing the equation of the function after the horizontal compression, because 9 is a perfect square, it was brought outside the square root sign as 3. So, the transformation can now be thought of as a vertical stretch by a factor of 3.

c) Write the equation of a different function whose image would be the same after two different stretches or compressions. Justify your answer.

Sample response: I chose the function $y = x^2$. The graph of $y = 4x^2$ is the image of the graph of $y = x^2$ after a vertical stretch by a factor of 4. The graph of $y = (2x)^2$, or $y = 4x^2$ is the image of the graph of $y = x^2$ after a horizontal compression by a factor of $\frac{1}{2}$.
So, the image of $y = x^2$ after a vertical stretch by a factor of 4 is the same as the image of $y = x^2$ after a horizontal compression by a factor of $\frac{1}{2}$.

C

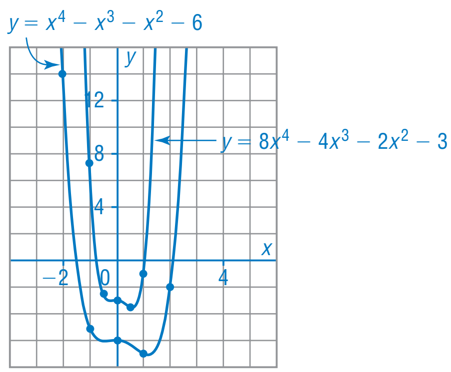
15. a) Write the equation of a quartic or quintic polynomial function.

Sample response: $y = x^4 - x^3 - x^2 - 6$

b) Sketch its graph.

The equation represents an even-degree polynomial function. Since the leading coefficient is positive, the graph opens up.
The constant term is −6, so the y-intercept is −6.
Use a table of values to create the graph.

x	y
−2	14
−1	−5
0	−6
1	−7
2	−2



c) Choose a vertical and a horizontal stretch or compression. Sketch the final image after these transformations on the grid in part b.

Sample response: I chose a horizontal compression by a factor of $\frac{1}{2}$ and a vertical compression by a factor of $\frac{1}{2}$.

Use: (x, y) on $y = x^4 - x^3 - x^2 - 6$ corresponds to $\left(\frac{x}{2}, \frac{1}{2}y\right)$.

(x, y)	$\left(\frac{x}{2}, \frac{1}{2}y\right)$
(−2, 14)	(−1, 7)
(−1, −5)	(−0.5, −2.5)
(0, −6)	(0, −3)
(1, −7)	(0.5, −3.5)
(2, −2)	(1, −1)

d) Write an equation of the final image.

Sample response: The graph of $y = x^4 - x^3 - x^2 - 6$ was horizontally compressed by a factor of $\frac{1}{2}$ and vertically compressed by a factor of $\frac{1}{2}$. So, $a = \frac{1}{2}$ and $b = 2$. To write the equation of the final image, replace x with $2x$ and multiply y by $\frac{1}{2}$:

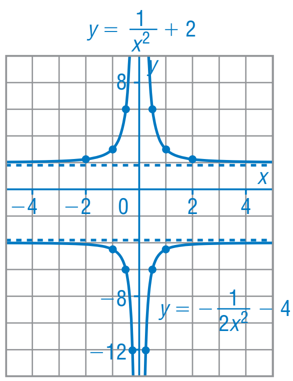
$y = \frac{1}{2}((2x)^4 - (2x)^3 - (2x)^2 - 6)$

$y = \frac{1}{2}(16x^4 - 8x^3 - 4x^2 - 6)$

$y = 8x^4 - 4x^3 - 2x^2 - 3$

16. On the same grid:

a) Sketch the graph of $y = \frac{1}{x^2} + 2$.



$y = \frac{1}{x^2} + 2$ is undefined when $x = 0$.

So, the line $x = 0$ is a vertical asymptote.

When $|x| \to \infty$, $\frac{1}{x^2} + 2 \to 2$

So, the line $y = 2$ is a horizontal asymptote.

Use a table of values to sketch the graph.

x	y
−2	2.25
−1	3
−0.5	6
0.5	6
1	3
2	2.25

b) Sketch the final image after a vertical stretch by a factor of 2, a reflection in the x-axis, and a horizontal compression by a factor of $\frac{1}{2}$.

The graph is vertically stretched by a factor of 2 and reflected in the x-axis, so $a = -2$. The graph is horizontally compressed by a factor of $\frac{1}{2}$, so $b = 2$. Use: (x, y) on $y = \frac{1}{x^2} + 2$ corresponds to $\left(\frac{x}{2}, -2y\right)$ on the final image.

(x, y)	$\left(\frac{x}{2}, -2y\right)$
$(-2, 2.25)$	$(-1, -4.5)$
$(-1, 3)$	$(-0.5, -6)$
$(-0.5, 6)$	$(-0.25, -12)$
$(0.5, 6)$	$(0.25, -12)$
$(1, 3)$	$(0.5, -6)$
$(2, 2.25)$	$(1, -4.5)$

c) How does the final image relate to the graph of $y = \frac{1}{x^2}$? Are the asymptotes the same? Explain.

The equation of the final image is
$$y = -2\left(\frac{1}{(2x)^2} + 2\right)$$
$$= \frac{-2}{4x^2} - 4$$
$$= \frac{-1}{2x^2} - 4$$

So, the final image is the graph of $y = \frac{1}{x^2}$ after a vertical compression by a factor of $\frac{1}{2}$, a reflection in the x-axis, and a translation of 4 units down. The vertical asymptotes are the same, but the equation of the horizontal asymptote is $y = -4$.