Lesson 4.2 Exercises, pages 278–284

A

3. Given $f(x) = x^3$ and $g(x) = x^2 + 1$, write an explicit equation for each combination.

a) $h(x) = f(x) + g(x)$

$h(x) = x^3 + x^2 + 1$

b) $d(x) = f(x) - g(x)$

$d(x) = x^3 - x^2 - 1$

c) $p(x) = f(x) \cdot g(x)$

$p(x) = x^3(x^2 + 1)$

d) $q(x) = \dfrac{f(x)}{g(x)}$

$q(x) = \dfrac{x^3}{x^2 + 1}$

4. For each function $h(x)$ below, write explicit equations for $f(x)$ and $g(x)$ so that:

i) $h(x)$ is the sum $f(x) + g(x)$

ii) $h(x)$ is the difference $f(x) - g(x)$

a) $h(x) = x^2 + 3x - 4$

Sample answers:

i) $h(x) = x^2 + (3x - 4)$
$f(x) = x^2$ and $g(x) = 3x - 4$

ii) $h(x) = x^2 - (-3x + 4)$
$f(x) = x^2$ and
$g(x) = -3x + 4$

b) $h(x) = x^3 - x^2 + 8$

i) $h(x) = (x^3 - x^2) + 8$
$f(x) = x^3 - x^2$ and $g(x) = 8$

ii) $h(x) = x^3 - (x^2 - 8)$
$f(x) = x^3$ and $g(x) = x^2 - 8$

B

5. Use $f(x) = 2x - 4$ and $g(x) = -x + 2$.

a) Write an explicit equation for $h(x)$.

i) $h(x) = f(x) + g(x)$

$h(x) = 2x - 4 + (-x + 2)$
$h(x) = x - 2$

ii) $h(x) = g(x) + f(x)$

$h(x) = -x + 2 + 2x - 4$
$h(x) = x - 2$

iii) $h(x) = f(x) - g(x)$

$h(x) = 2x - 4 - (-x + 2)$
$h(x) = 3x - 6$

iv) $h(x) = g(x) - f(x)$

$h(x) = -x + 2 - (2x - 4)$
$h(x) = -3x + 6$

v) $h(x) = f(x) \cdot g(x)$

$h(x) = (2x - 4)(-x + 2)$
$h(x) = -2x^2 + 8x - 8$

vi) $h(x) = g(x) \cdot f(x)$

$h(x) = (-x + 2)(2x - 4)$
$h(x) = -2x^2 + 8x - 8$

b) For part a, compare the answers to parts i and ii; parts iii and iv; and parts v and vi. Explain the results.

The answers to parts i and ii are the same because addition is commutative. The answers to parts iii and iv are opposites because subtraction is not commutative. The answers to parts v and vi are the same because multiplication is commutative.

6. Given that $f(x) = x^2 - 4$, $g(x) = 2x - 1$, and $h(x) = 3 - x^3$, write an explicit equation for $k(x)$, then state its domain.

a) $k(x) = f(x) + g(x) + h(x)$

$k(x) = x^2 - 4 + 2x - 1 + 3 - x^3$
$k(x) = -x^3 + x^2 + 2x - 2$
This is a cubic function; its domain is: $x \in \mathbb{R}$

b) $k(x) = f(x) - g(x) + h(x)$

$k(x) = x^2 - 4 - (2x - 1) + 3 - x^3$
$k(x) = -x^3 + x^2 - 2x$
This is a cubic function; its domain is: $x \in \mathbb{R}$

c) $k(x) = f(x) + g(x) \cdot h(x)$

$k(x) = x^2 - 4 + (2x - 1)(3 - x^3)$
$k(x) = x^2 - 4 + 6x - 2x^4 - 3 + x^3$
$k(x) = -2x^4 + x^3 + x^2 + 6x - 7$
This is a quartic function; its domain is: $x \in \mathbb{R}$

d) $k(x) = g(x) \cdot f(x) - h(x)$

$k(x) = (2x - 1)(x^2 - 4) - (3 - x^3)$
$k(x) = 2x^3 - 8x - x^2 + 4 - 3 + x^3$
$k(x) = 3x^3 - x^2 - 8x + 1$
This is a cubic function; its domain is: $x \in \mathbb{R}$

7. Use the function $k(x) = x^2 - 3x - 28$.

a) Write explicit equations for three functions $f(x)$, $g(x)$, and $h(x)$ so that $k(x) = f(x) + g(x) + h(x)$.

Sample response:
$k(x) = x^2 - 3x - 28$
$k(x) = (x^2) + (-3x) + (-28)$
$f(x) = x^2$; $g(x) = -3x$; $h(x) = -28$

b) Write explicit equations for two functions $f(x)$ and $g(x)$ so that $k(x) = f(x) \cdot g(x)$.

Sample response:
$k(x) = x^2 - 3x - 28$
Factor: $k(x) = (x - 7)(x + 4)$
$f(x) = x - 7$; $g(x) = x + 4$

8. For each function $h(x)$ below, write explicit equations for $f(x)$ and $g(x)$ so that:

i) $h(x)$ is the sum $f(x) + g(x)$

ii) $h(x)$ is the difference $f(x) - g(x)$

iii) $h(x)$ is the product $f(x) \cdot g(x)$

iv) $h(x)$ is the quotient $\frac{f(x)}{g(x)}$

a) $h(x) = x^2$

Sample response:

i) Subtract and add the same term.
$h(x) = x^2 - x + x$
$f(x) = x^2 - x$; $g(x) = x$

ii) Add and subtract the same term.
$h(x) = x^2 + x - x$
$f(x) = x^2 + x$; $g(x) = x$

iii) Write x^2 as a product.
$h(x) = (x)(x)$
$f(x) = x$; $g(x) = x$

iv) Multiply and divide x^2 by the same non-zero expression.
$h(x) = \frac{x^2(x^2 + 1)}{x^2 + 1}$
$f(x) = x^2(x^2 + 1)$;
$g(x) = x^2 + 1$

b) $h(x) = \sqrt{x}$

i) Subtract and add the same term.
$h(x) = \sqrt{x} - x + x$
$f(x) = \sqrt{x} - x$; $g(x) = x$

ii) Add and subtract the same term.
$h(x) = \sqrt{x} + x - x$
$f(x) = \sqrt{x} + x$; $g(x) = x$

iii) Multiply $\sqrt{x}$ by a term that is equal to 1.
$h(x) = \sqrt{x}\left(\frac{2}{2}\right)$
$f(x) = 2\sqrt{x}$; $g(x) = \frac{1}{2}$

iv) Multiply and divide $\sqrt{x}$ by the same non-zero expression.
$h(x) = \sqrt{x}\left(\frac{2 + x^2}{2 + x^2}\right)$
$f(x) = (2 + x^2)\sqrt{x}$;
$g(x) = 2 + x^2$

9. Use $f(x) = |x - 4|$ and $g(x) = x^2$.

a) State the domain and range of $f(x)$ and of $g(x)$.

$f(x)$ is an absolute value function; the domain is $x \in \mathbb{R}$ and the range is $y \geq 0$. $g(x)$ is a quadratic function whose graph has vertex (0, 0) and opens up; the domain is $x \in \mathbb{R}$, and the range is $y \geq 0$.

b) Given $h(x) = f(x) + g(x)$, write an explicit equation for $h(x)$, then determine its domain and range.

$h(x) = |x - 4| + x^2$
Since the domains of $f(x)$ and $g(x)$ are equal, then the domain of $h(x)$ is $x \in \mathbb{R}$. Use technology to graph the function; the minimum value is 3.75 at $x = 0.5$, so the range is $y \geq 3.75$.

c) Given $d(x) = f(x) - g(x)$, write an explicit equation for $d(x)$, then determine its domain and range.

$d(x) = |x - 4| - x^2$
Since the domains of $f(x)$ and $g(x)$ are equal, then the domain of $d(x)$ is $x \in \mathbb{R}$. Use technology to graph the function; the maximum value is 4.25 at $x = -0.5$, so the range is $y \leq 4.25$.

10. Use $f(x) = x^3 - x$ and $g(x) = \dfrac{1}{x + 3}$.

a) State the domain and range of $f(x)$ and of $g(x)$.

$f(x)$ is a cubic function; the domain is $x \in \mathbb{R}$ and the range is $y \in \mathbb{R}$. $g(x)$ is a reciprocal function; the domain is $x \neq -3$, and the range is $y \neq 0$.

b) Given $h(x) = f(x) + g(x)$, write an explicit equation for $h(x)$, then determine its domain and range.

$h(x) = x^3 - x + \dfrac{1}{x + 3}$
The domain of $h(x)$ is the set of values of x that are common to the domains of $f(x)$ and $g(x)$, so the domain is $x \neq -3$. Use technology to graph the function; the approximate range is $y \leq -34.5$ or $y \geq -14.2$.

c) Given $p(x) = f(x) \cdot g(x)$, write an explicit equation for $p(x)$, then determine its domain and range.

$p(x) = \dfrac{x^3 - x}{x + 3}$
The domain of $p(x)$ is the set of values of x that are common to the domains of $f(x)$ and $g(x)$, so the domain is $x \neq -3$. Use technology to graph the function; the range is $y \in \mathbb{R}$.

11. Use $f(x) = \sqrt{x + 2}$ and $g(x) = |x - 2|$.

a) State the domain and range of $f(x)$ and of $g(x)$.

$f(x)$ is a square root function; the domain is $x \geq -2$ and the range is $y \geq 0$. $g(x)$ is an absolute value function; the domain is $x \in \mathbb{R}$, and the range is $y \geq 0$.

b) Given $p(x) = f(x) \cdot g(x)$, write an explicit equation for $p(x)$, then determine its domain and range.

$p(x) = \sqrt{x + 2} \cdot |x - 2|$
The domain of $p(x)$ is all values of x that are common to the domains of $f(x)$ and $g(x)$, so the domain is $x \geq -2$. Use technology to graph the function; the range is $y \geq 0$.

c) Given $q(x) = \frac{f(x)}{g(x)}$, write an explicit equation for $q(x)$, then determine its domain and range.

$q(x) = \frac{\sqrt{x + 2}}{|x - 2|}$

The domain of $q(x)$ is restricted to those values of x for which $|x - 2| \neq 0$ and for which $\sqrt{x + 2}$ is defined, so the domain is $x \geq -2$, $x \neq 2$. Use technology to graph the function; the range is $y \geq 0$.

12. a) When asked to write $f(x) = x^2$ as the quotient of two functions, a student wrote $f(x) = \frac{x^3}{x}$. Is this correct? Justify your answer.

No, the answer is incorrect because the domain of the new function has the restriction $x \neq 0$, which the original function did not have.

b) If your answer to part a is no, write $f(x) = x^2$ as a quotient of two functions.

Multiply and divide the function by a non-zero expression, such as $(x^2 + 1)$.

A possible function is: $f(x) = \frac{x^2(x^2 + 1)}{x^2 + 1}$

13. Consider the functions: $f(x) = (x + 3)^2$ and $g(x) = \frac{x - 2}{x + 3}$

Given $p(x) = f(x) \cdot g(x)$, write an explicit equation for $p(x)$, then determine its domain and range.

$p(x) = (x + 3)^2\left(\frac{x - 2}{x + 3}\right)$, or $p(x) = (x + 3)(x - 2), x \neq -3$

$p(x) = x^2 + x - 6, x \neq -3$

The domain is $x \neq -3$. Use technology to graph the function; the range is $y \geq -6.25$.

14. Consider the function $g(x) = 4$ and any function $f(x)$. Predict how the graph of each function below will be a transformation image of $y = f(x)$. Use graphing technology to check.

a) $y = f(x) + g(x)$

b) $y = f(x) - g(x)$

The function $g(x)$ is a horizontal line with y-intercept 4.

a) When $g(x)$ is added to $f(x)$, the graph of $y = f(x)$ will be translated 4 units up.

b) When $g(x)$ is subtracted from $f(x)$, the graph of $y = f(x)$ will be translated 4 units down.

c) $y = f(x) \cdot g(x)$

When $f(x)$ is multiplied by $g(x)$, the graph of $y = f(x)$ will be stretched vertically by a factor of 4.

d) $y = \frac{f(x)}{g(x)}$

When $f(x)$ is divided by $g(x)$, the graph of $y = f(x)$ will be compressed vertically by a factor of $\frac{1}{4}$.

15. When each function $h(x)$ below is evaluated at $x = a$, its value is 0. What do you know about the values of $f(a)$ and $g(a)$?

a) $h(x) = f(x) + g(x)$

Substitute: $x = a$, $h(a) = 0$
$0 = f(a) + g(a)$
$f(a) = -g(a)$

b) $h(x) = f(x) - g(x)$

Substitute: $x = a$, $h(a) = 0$
$0 = f(a) - g(a)$
$f(a) = g(a)$

c) $h(x) = f(x) \cdot g(x)$

Substitute: $x = a$, $h(a) = 0$
$0 = f(a) \cdot g(a)$
$f(a) = 0$, or $g(a) = 0$, or both

d) $h(x) = \dfrac{f(x)}{g(x)}$

Substitute: $x = a$, $h(a) = 0$
$0 = \dfrac{f(a)}{g(a)}$
$f(a) = 0$ and $g(a) \neq 0$

16. Given $f(x) = \sqrt{x}$ and $g(x) = \sqrt{2 - x}$, determine an explicit equation for each function, then state its domain.

a) $h(x) = f(x) + g(x)$

$h(x) = \sqrt{x} + \sqrt{2 - x}$
For $f(x)$, $x \geq 0$ and for $g(x)$, $x \leq 2$, so the domain of $h(x)$ is: $0 \leq x \leq 2$

b) $d(x) = f(x) - g(x)$

$d(x) = \sqrt{x} - \sqrt{2 - x}$
For $f(x)$, $x \geq 0$ and for $g(x)$, $x \leq 2$, so the domain of $d(x)$ is: $0 \leq x \leq 2$

c) $p(x) = f(x) \cdot g(x)$

$p(x) = \sqrt{x} \cdot \sqrt{2 - x}$
For $f(x)$, $x \geq 0$ and for $g(x)$, $x \leq 2$, so the domain of $p(x)$ is: $0 \leq x \leq 2$

d) $q(x) = \dfrac{f(x)}{g(x)}$

$q(x) = \dfrac{\sqrt{x}}{\sqrt{2 - x}}$
For $f(x)$, $x \geq 0$ and for $g(x)$, $x \leq 2$, but since $g(x)$ is in the denominator, $x \neq 2$
The domain of $q(x)$ is: $0 \leq x < 2$

C

17. Consider the function: $f(x) = \dfrac{x^2 - 3x + 4}{x - 1}$

a) Determine the domain and the approximate range of $f(x)$.

Since the denominator cannot be 0, the domain is: $x \neq 1$
Use technology to graph the function.
It has a minimum point at approximately (2.4, 1.8) and a maximum point at approximately (−0.4, −3.8).
So, the range is approximately $y \leq -3.8$ or $y \geq 1.8$.

b) Determine explicit equations for $g(x)$, $h(x)$, and $k(x)$ so that

$$f(x) = g(x) + \frac{h(x)}{k(x)}.$$

Sample response: Use synthetic division to determine: $(x^2 - 3x + 4) \div (x - 1)$

$$\begin{array}{r|rrr} 1 & 1 & -3 & 4 \\ & & 1 & -2 \\ \hline & 1 & -2 & 2 \end{array}$$

The function can be written as:

$$f(x) = x - 2 + \frac{2}{x - 1}$$

So, $g(x) = x - 2$; $h(x) = 2$; and $k(x) = x - 1$

18. Is it possible to combine $f(x) = \sqrt{x}$ with a second function $g(x)$ to get a new function whose domain is all real numbers? Justify your answer.

No, when two functions are combined, the domain of the new function is the set of values of x that are common to the two functions that were combined. Since the domain of $\sqrt{x}$ is $x \geq 0$, then the domain of the new function cannot be all real numbers.