PRACTICE TEST, pages 335–338

1. **Multiple Choice** Given $f(x) = \sqrt{x}$ and $g(x) = 3x - 6$, which function is a composition of f and g?

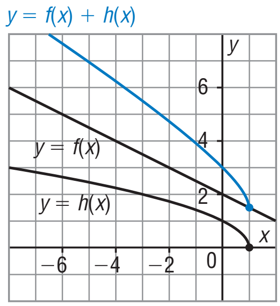
A. $y = \sqrt{x} + 3x - 6$ [✓ **B.**] $y = 3\sqrt{x} - 6$

C. $y = \dfrac{\sqrt{x}}{3x - 6}$ **D.** $y = \sqrt{x} - 3x + 6$

2. **Multiple Choice** For $f(x) = 3x - 5$ and $g(x) = 4x^2 - 7$, which value is greatest?

A. $f(g(1))$ **B.** $g(f(1))$ **C.** $f(f(1))$ [✓ **D.**] $g(g(1))$

3. Use the graphs of $y = f(x)$ and $y = h(x)$ to graph $y = f(x) + h(x)$.



From the graphs:

x	$f(x)$	$h(x)$	$f(x) + h(x)$
1	1.5	0	1.5
0	2	1	3
−2	3	≐ 1.7	≐ 4.7
−3	3.5	2	5.5
−4	4	≐ 2.2	≐ 6.2
−6	5	≐ 2.6	≐ 7.6
−8	6	3	9

Plot the points with coordinates $(x, f(x) + h(x))$, then join the points with a smooth curve.

Use these functions for questions 4 to 9:

$f(x) = 2 - 0.5x \quad g(x) = x^2 + 5x \quad h(x) = \sqrt{1 - x} \quad k(x) = \frac{1}{4 - x}$

4. Write an explicit equation for each combination of functions, then determine the domain and range of the function. Approximate the range where necessary.

a) $y = g(x) \cdot f(x)$

$y = (x^2 + 5x)(2 - 0.5x)$

$y = 2x^2 - 0.5x^3 + 10x - 2.5x^2$

$y = -0.5x^3 - 0.5x^2 + 10x$

This is a cubic function; its domain is: $x \in \mathbb{R}$; and its range is: $y \in \mathbb{R}$

b) $y = \frac{f(x)}{h(x)}$

$y = \frac{2 - 0.5x}{\sqrt{1 - x}}$

Since $1 - x \geq 0$, then $x \leq 1$

The domain is: $x \leq 1$

Use graphing technology. From the graph, the approximate range is: $y \geq 1.7$

5. Write explicit equations in each case.

a) two functions $a(x)$ and $b(x)$ so that $g(x) = a(x) \cdot b(x)$

Sample response:

Factor: $g(x) = x^2 + 5x$

$g(x) = x(x + 5)$

So, $a(x) = x$ and $b(x) = x + 5$

b) three functions $a(x)$, $b(x)$, and $c(x)$ so that $f(x) = a(x) + b(x) + c(x)$

Sample response:

$f(x) = 2 - 0.5x$

Write $-0.5x$ as the sum of two terms.

$f(x) = 2 - x + 0.5x$

$f(x) = 2 + (-x) + 0.5x$

So, $a(x) = 2$, $b(x) = -x$, and $c(x) = 0.5x$

c) i) two functions $a(x)$ and $b(x)$ so that $k(x) = a(b(x))$

Sample response:

$k(x) = \frac{1}{4 - x}$

$b(x) = 4 - x$ and $a(x) = \frac{1}{x}$

ii) three functions $a(x)$, $b(x)$, and $c(x)$ so that $k(x) = a(b(c(x)))$

Sample response:

$k(x) = \frac{1}{4 - x}$

$c(x) = -x$, $b(x) = 4 + x$, and $a(x) = \frac{1}{x}$

6. Determine each value.

a) $f(g(2))$

$g(x) = x^2 + 5x$ and
$f(x) = 2 - 0.5x$
$g(2) = 2^2 + 5(2)$
$g(2) = 14$
So, $f(g(2)) = f(14)$
$= 2 - 0.5(14)$
$= -5$

b) $g(h(-4))$

$h(x) = \sqrt{1 - x}$
and $g(x) = x^2 + 5x$
$h(-4) = \sqrt{1 - (-4)}$
$h(-4) = \sqrt{5}$
So, $g(h(-4)) = g(\sqrt{5})$
$= (\sqrt{5})^2 + 5\sqrt{5}$
$= 5 + 5\sqrt{5}$

7. Determine an explicit equation for each composite function and explain any restrictions on x.

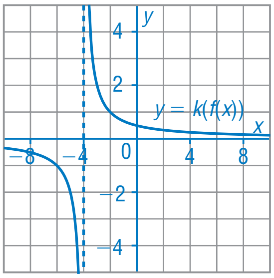
a) $g(h(x))$

$h(x) = \sqrt{1 - x}$ and
$g(x) = x^2 + 5x$
In $g(x) = x^2 + 5x$, replace
x with $\sqrt{1 - x}$.
$g(h(x)) = (\sqrt{1 - x})^2 + 5\sqrt{1 - x}$
Since the square root of a real number cannot be negative,
$1 - x \geq 0$, so $x \leq 1$
So, $g(h(x)) = 1 - x + 5\sqrt{1 - x}$

b) $h(f(x))$

$f(x) = 2 - 0.5x$ and
$h(x) = \sqrt{1 - x}$
In $h(x) = \sqrt{1 - x}$, replace x with
$2 - 0.5x$.
$h(f(x)) = \sqrt{1 - (2 - 0.5x)}$
$h(f(x)) = \sqrt{-1 + 0.5x}$
Since the square root of a real number cannot be negative,
$-1 + 0.5x \geq 0$, so $x \geq 2$

8. Sketch a graph of the composite function $y = k(f(x))$, then state the domain of the composite function.



$f(x) = 2 - 0.5x$ and $k(x) = \dfrac{1}{4 - x}$

The domain of $f(x)$ is $x \in \mathbb{R}$, and the domain of $k(x)$ is $x \neq 4$.

For $k(f(x))$, replace x in $k(x) = \dfrac{1}{4 - x}$ with $2 - 0.5x$.

$$k(f(x)) = \frac{1}{4 - (2 - 0.5x)}$$

$$k(f(x)) = \frac{1}{2 + 0.5x}$$

For $k(f(x))$: $2 + 0.5x \neq 0$, or $x \neq -4$
So, the domain of $k(f(x))$ is: $x \neq -4$

$k(f(x)) = \dfrac{1}{2 + 0.5x}$ is a reciprocal function; it has a vertical asymptote with equation $x = -4$, and a horizontal asymptote with equation $y = 0$.
Make a table of values, then join the plotted points with 2 smooth curves.

x	-6	-5	-3	-2	0	2	6
$k(f(x))$	-1	-2	2	1	0.5	$0.\overline{3}$	0.2

9. Write an explicit equation for each combination.

a) $y = k(x) + h(g(x))$

$g(x) = x^2 + 5x$ and
$h(x) = \sqrt{1 - x}$
$h(g(x)) = \sqrt{1 - (x^2 + 5x)}$
$h(g(x)) = \sqrt{1 - x^2 - 5x}$
So, $y = \frac{1}{4 - x} + \sqrt{1 - x^2 - 5x}$

b) $y = f(f(x)) - g(g(x))$

$f(x) = 2 - 0.5x$
$f(f(x)) = 2 - 0.5(2 - 0.5x)$
$f(f(x)) = 1 + 0.25x$
$g(g(x)) = (x^2 + 5x)^2 + 5(x^2 + 5x)$
$g(g(x)) = x^4 + 10x^3 + 25x^2 + 5x^2 + 25x$
$g(g(x)) = x^4 + 10x^3 + 30x^2 + 25x$
So, $y = 1 + 0.25x - (x^4 + 10x^3 + 30x^2 + 25x)$
$y = 1 - 24.75x - x^4 - 10x^3 - 30x^2$