# Lesson 5.3 Exercises, pages 364–368

## A

**3.** Write each number as a power of 2.

a) 16 $= 2^4$

b) 128 $= 2^7$

c) $\frac{1}{32}$ $= 2^{-5}$

d) 1 $= 2^0$

**4.** Which numbers below can be written as powers of 5? Write each number you identify as a power of 5.

a) 125 $= 5^3$

b) 10 cannot be written as a power of 5

c) $\frac{1}{25}$ $= 5^{-2}$

d) 1 $= 5^0$

**5.** Write each number as a power of 3.

a) $\sqrt[3]{9}$

$= 9^{\frac{1}{3}}$
$= (3^2)^{\frac{1}{3}}$
$= 3^{\frac{2}{3}}$

b) $\sqrt{243}$

$= 243^{\frac{1}{2}}$
$= (3^5)^{\frac{1}{2}}$
$= 3^{\frac{5}{2}}$

c) $\frac{\sqrt{3}}{3}$

$= 3^{\frac{1}{2}} \cdot 3^{-1}$
$= 3^{\frac{1}{2}-1}$
$= 3^{-\frac{1}{2}}$

d) $27\sqrt{3}$

$= 3^3 \cdot 3^{\frac{1}{2}}$
$= 3^{3+\frac{1}{2}}$
$= 3^{\frac{7}{2}}$

## B

**6.** Solve each equation.

a) $2^x = 256$

$2^x = 2^8$
$x = 8$

b) $81 = 3^{x+1}$

$3^4 = 3^{x+1}$
$4 = x + 1$
$x = 3$

c) $3^x = 9^{x-2}$

$3^x = (3^2)^{x-2}$
$x = 2(x - 2)$
$x = 2x - 4$
$x = 4$

d) $4^{x-1} = 2^{x+3}$

$(2^2)^{x-1} = 2^{x+3}$
$2(x - 1) = x + 3$
$2x - 2 = x + 3$
$x = 5$

e) $8^{2x} = 16^{x+3}$

$(2^3)^{2x} = (2^4)^{x+3}$
$3(2x) = 4(x + 3)$
$6x = 4x + 12$
$2x = 12$
$x = 6$

f) $9^{x+1} = 243^{x+3}$

$(3^2)^{x+1} = (3^5)^{x+3}$
$2(x + 1) = 5(x + 3)$
$2x + 2 = 5x + 15$
$-3x = 13$
$x = -\frac{13}{3}$

**7.** Use graphing technology to solve each equation. Give the solution to the nearest tenth.

a) $10 = 2^x$

**Graph: $y = 2^x - 10$**
**The approximate zero is 3.3219281**
**$x \doteq 3.3$**

b) $3^x = 100$

**Graph: $y = 100 - 3^x$**
**The approximate zero is 4.1918065**
**$x \doteq 4.2$**

c) $3^{x+1} = 50$

**Graph: $y = 50 - 3^{x+1}$**
**The approximate zero is 2.5608768**
**$x \doteq 2.6$**

d) $30 = 2^{x-1}$

**Graph: $y = 2^{x-1} - 30$**
**The approximate zero is 5.9068906**
**$x \doteq 5.9$**

**8.** Explain why the equation $4^x = -2$ does not have a real solution. Verify, graphically, that there is no solution.

**The value of a power with a positive base can never be negative, so the equation does not have a real solution. When I graph $y = -2 - 4^x$, the graph does not have an $x$-intercept.**

**9.** Solve each equation.

a) $2^x = 8\sqrt[3]{2}$

$$2^x = 2^3 \cdot 2^{\frac{1}{3}}$$
$$2^x = 2^{3+\frac{1}{3}}$$
$$x = 3 + \frac{1}{3}$$
$$x = \frac{10}{3}$$

b) $81\sqrt{3} = 3^x$

$$3^4 \cdot 3^{\frac{1}{2}} = 3^x$$
$$3^{4+\frac{1}{2}} = 3^x$$
$$4 + \frac{1}{2} = x$$
$$x = \frac{9}{2}$$

c) $2^{x+1} = 2\sqrt[3]{4}$

$$2^{x+1} = 2 \cdot 4^{\frac{1}{3}}$$
$$2^{x+1} = 2(2^2)^{\frac{1}{3}}$$
$$2^{x+1} = 2^1 \cdot 2^{\frac{2}{3}}$$
$$2^{x+1} = 2^{1+\frac{2}{3}}$$
$$x + 1 = 1 + \frac{2}{3}$$
$$x = \frac{2}{3}$$

d) $9^x = \sqrt{27}$

$$(3^2)^x = (3^3)^{\frac{1}{2}}$$
$$2x = \frac{3}{2}$$
$$x = \frac{3}{4}$$

e) $\sqrt[4]{216} = 36^{x-1}$

$$216^{\frac{1}{4}} = 6^{2(x-1)}$$
$$(6^3)^{\frac{1}{4}} = 6^{2(x-1)}$$
$$\frac{3}{4} = 2x - 2$$
$$2x = \frac{11}{4}$$
$$x = \frac{11}{8}$$

f) $(\sqrt{7})^{x+1} = \sqrt[3]{49}$

$$7^{\frac{1}{2}(x+1)} = (7^2)^{\frac{1}{3}}$$
$$\frac{1}{2}x + \frac{1}{2} = \frac{2}{3}$$
$$3x + 3 = 4$$
$$3x = 1$$
$$x = \frac{1}{3}$$

**10.** Solve each equation.

a) $\left(\frac{1}{4}\right)^3 = 2^x$

$(2^{-2})^3 = 2^x$
$(-2)(3) = x$
$x = -6$

b) $5^x = \frac{\sqrt[3]{25}}{25}$

$5^x = (5^2)^{\frac{1}{3}} \cdot 5^{-2}$
$5^x = 5^{\frac{2}{3}-2}$
$x = \frac{2}{3} - 2$
$x = -\frac{4}{3}$

c) $\frac{\sqrt[3]{49}}{343} = 7^{x+1}$

$(7^2)^{\frac{1}{3}} \cdot 7^{-3} = 7^{x+1}$
$7^{\frac{2}{3}-3} = 7^{x+1}$
$\frac{2}{3} - 3 = x + 1$
$-\frac{7}{3} = x + 1$
$x = -\frac{10}{3}$

d) $\left(\frac{1}{9}\right)^x = 3\sqrt{27}$

$(3^{-2})^x = 3^1 \cdot (3^3)^{\frac{1}{2}}$
$3^{-2x} = 3^{1+\frac{3}{2}}$
$-2x = 1 + \frac{3}{2}$
$-2x = \frac{5}{2}$
$x = -\frac{5}{4}$

e) $8^{1-x} = \frac{\sqrt[3]{16}}{4}$

$2^{3(1-x)} = (2^4)^{\frac{1}{3}} \cdot 2^{-2}$
$2^{3(1-x)} = 2^{\frac{4}{3}-2}$
$3 - 3x = \frac{4}{3} - 2$
$-3x = -\frac{11}{3}$
$x = \frac{11}{9}$

f) $\left(\frac{1}{8}\right)^{x+1} = (\sqrt[3]{16})^x$

$(2^{-3})^{x+1} = \left((2^4)^{\frac{1}{3}}\right)^x$
$-3x - 3 = \frac{4}{3}x$
$-\frac{13}{3}x = 3$
$x = -\frac{9}{13}$

**11.** Use graphing technology to solve each equation. Give the solution to the nearest tenth.

a) $2 = 1.05^x$

Graph: $y = 1.05^x - 2$
The approximate zero is 14.206699
$x \doteq 14.2$

b) $2^{-\frac{x}{5}} = 0.4$

Graph: $y = 0.4 - 2^{-\frac{x}{5}}$
The approximate zero is 6.6096405
$x \doteq 6.6$

c) $2^{x+1} = 3^{x-2}$

Graph: $y = 3^{x-2} - 2^{x+1}$
The approximate zero is 7.1285339
$x \doteq 7.1$

d) $3(2^x) = 64$

Graph: $y = 64 - 3(2^x)$
The approximate zero is 4.4150375
$x \doteq 4.4$

**12.** A principal of \$600 was invested in a term deposit that pays 5.5% annual interest, compounded semi-annually. To the nearest tenth of a year, when will the amount be \$1000?

Use: $A = A_0\left(1 + \frac{i}{n}\right)^{nt}$ Substitute: $A = 1000, A_0 = 600, i = 0.055, n = 2$

$$1000 = 600\left(1 + \frac{0.055}{2}\right)^{2t}$$

Graph $y = 600\left(1 + \frac{0.055}{2}\right)^{2t} - 1000$, then determine the zero of the function.

The approximate zero is 9.4148676

It will take approximately 9.4 years for the term deposit to amount to \$1000.

**13.** **a)** To the nearest year, how long will it take an investment of \$500 to double at each annual interest rate, compounded annually?

**i)** 4% **ii)** 6% **iii)** 8%

**iv)** 9% **v)** 12%

Use: $A = A_0\left(1 + \frac{i}{n}\right)^{nt}$ Substitute: $A = 1000, A_0 = 500, n = 1$

$$1000 = 500\left(1 + \frac{i}{1}\right)^{1t}$$

$2 = (1 + i)^t$ Use this expression below.

i) Substitute: $i = 0.04$

$2 = (1 + 0.04)^t$

$2 = 1.04^t$

Graph $y = 1.04^t - 2$, then determine the zero of the function.

The approximate zero is: 17.672988

It will take approximately 18 years.

ii) Substitute: $i = 0.06$

$2 = (1 + 0.06)^t$

$2 = 1.06^t$

Graph: $y = 1.06^t - 2$

The approximate zero is: 11.895661

It will take approximately 12 years.

iii) Substitute: $i = 0.08$

$2 = (1 + 0.08)^t$

$2 = 1.08^t$

Graph: $y = 1.08^t - 2$

The approximate zero is: 9.0064683

It will take approximately 9 years.

iv) Substitute: $i = 0.09$

$2 = (1 + 0.09)^t$

$2 = 1.09^t$

Graph: $y = 1.09^t - 2$

The approximate zero is: 8.0432317

It will take approximately 8 years.

v) Substitute: $i = 0.12$

$2 = (1 + 0.12)^t$

$2 = 1.12^t$

Graph: $y = 1.12^t - 2$

The approximate zero is: 6.1162554

It will take approximately 6 years.

**b)** What pattern is there in the interest rates and times in part a?

The product of each interest rate as a percent and time in years is 72.

**14.** When light passes through glass, the intensity is reduced by 5%.

**a)** Determine a function that models the percent of light, $P$, that passes through $n$ layers of glass.

For 0 layers of glass, the percent of light is: $P = 100$
For 1 layer of glass, the percent of light is: $P = 100(0.95)$
For 2 layers of glass, the percent of light is: $P = 100(0.95)^2$
For 3 layers of glass, the percent of light is: $P = 100(0.95)^3$
For $n$ layers of glass, the percent of light is: $P = 100(0.95)^n$

**b)** Determine how many layers of glass are needed for only 25% of light to pass through.

Solve the equation: $25 = 100(0.95)^n$
Graph a related function: $y = 100(0.95)^x - 25$
The approximate zero of the function is: 27.026815
So, 27 layers of glass are needed.

**C**

**15.** Solve each equation, then verify the solution graphically.

**a)** $2^{(x^2)} = 16$

$2^{(x^2)} = 2^4$
$x^2 = 4$
$x = \pm 2$
The graph of $y = 16 - 2^{(x^2)}$ has $x$-intercepts 2 and $-2$.

**b)** $9^{x+4} = 3^{(x^2)}$

$3^{2(x+4)} = 3^{(x^2)}$
$2x + 8 = x^2$
$x^2 - 2x - 8 = 0$
$(x - 4)(x + 2) = 0$
$x = 4$ or $x = -2$
A graph of $y = 3^{(x^2)} - 9^{x+4}$ has $x$-intercepts $-2$ and 4.

**16.** For what values of $k$ does the equation $9^{(x^2)} = 27^{x+k}$ have no real solution?

$9^{(x^2)} = 27^{x+k}$
$3^{(2x^2)} = 3^{3(x+k)}$
$2x^2 = 3x + 3k$
$2x^2 - 3x - 3k = 0$
For no real roots, the discriminant is less than 0.
$(-3)^2 - 4(2)(-3k) < 0$
$(-3)^2 < 4(2)(-3k)$
$9 < -24k$
$k < -\frac{9}{24}$, or $-\frac{3}{8}$
The equation has no real solution when $k < -\frac{3}{8}$.