Lesson 5.6 Exercises, pages 405–410

A

3. Approximate the value of each logarithm, to the nearest thousandth.

a) $\log_2 9$ **b)** $\log_2 100$

Use the change of base formula to change the base of the logarithms to base 10.

$$\log_2 9 = \frac{\log 9}{\log 2} = 3.1699... \doteq 3.170$$

$$\log_2 100 = \frac{\log 100}{\log 2} = 6.6438... \doteq 6.644$$

4. Order these logarithms from greatest to least:
$\log_2 80$, $\log_3 900$, $\log_4 5000$, $\log_5 10\,000$

Write each logarithm to base 10, then calculate its value.

$$\log_2 80 = \frac{\log 80}{\log 2} = 6.3219...$$

$$\log_3 900 = \frac{\log 900}{\log 3} = 6.1918...$$

$$\log_4 5000 = \frac{\log 5000}{\log 4} = 6.1438...$$

$$\log_5 10\,000 = \frac{\log 10\,000}{\log 5} = 5.7227...$$

From greatest to least: $\log_2 80$, $\log_3 900$, $\log_4 5000$, $\log_5 10\,000$

B

5. Approximate the value of each logarithm, to the nearest thousandth. Write the related exponential expression.

a) $\log_7 400$

$$\log_7 400 = \frac{\log 400}{\log 7} = 3.0790... \doteq 3.079$$

So, $400 \doteq 7^{3.079}$

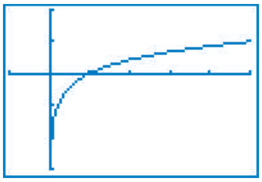
b) $\log_3\left(\frac{1}{2}\right)$

$$\log_3\left(\frac{1}{2}\right) = \frac{\log 0.5}{\log 3} = -0.6309... \doteq -0.631$$

So, $\frac{1}{2} \doteq 3^{-0.631}$

6. a) Use technology to graph $y = \log_5 x$. Sketch the graph.

Graph: $y = \frac{\log x}{\log 5}$



b) Identify the intercepts and the equation of the asymptote of the graph, and the domain and range of the function.

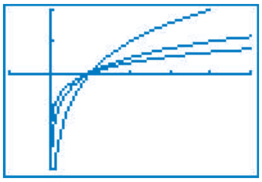
From the graph, the x-intercept is 1. There is no y-intercept. The equation of the asymptote is $x = 0$. The domain of the function is $x > 0$. The range of the function is $y \in \mathbb{R}$.

c) Choose the coordinates of two points on the graph. Multiply their x-coordinates and add their y-coordinates. What do you notice about the new coordinates? Explain the result.

From the TABLE, two points on the graph have coordinates:
(5, 1) and (25, 2)
The product of the x-coordinates is 125. The sum of the y-coordinates is 3.
The new coordinates are (125, 3), which is also a point on the graph.
The logarithm of the product of two numbers is the sum of the logarithms of the numbers.

7. a) Use a graphing calculator to graph $y = \log_2 x$, $y = \log_4 x$, and $y = \log_8 x$. Sketch the graphs.

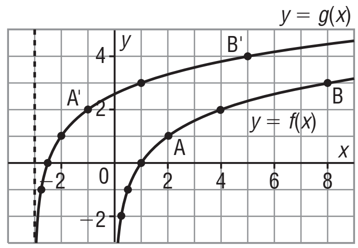
Graph: $y = \frac{\log x}{\log 2}$, $y = \frac{\log x}{\log 4}$, and $y = \frac{\log x}{\log 8}$



b) In part a, what happened to the graph of $y = \log_b x$, $b > 0$, $b \neq 1$, as the base changed?

As b increases, from $b = 2$, the graph of $y = \log_b x$ is compressed vertically by a factor of: $\dfrac{\frac{\log x}{\log b}}{\frac{\log x}{\log 2}} = \dfrac{\log 2}{\log b}$.

8. a) The graphs of a logarithmic function and its transformation image are shown. The functions are related by translations, and corresponding points are indicated. Identify the translations.



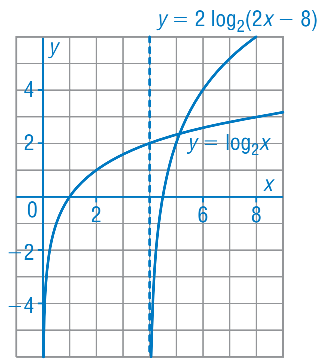
From A to A′, the translations are
3 units left and 1 unit up.
The same translations relate B and B′.

b) Given that $f(x) = \log_2 x$, what is $g(x)$? Justify your answer.

After translations, the image of the graph of $y = \log_2 x$ has equation:
$y - k = \log_2(x - h)$ Substitute: $k = 1$ and $h = -3$
The image graph has equation $y - 1 = \log_2(x + 3)$; or
$y = \log_2(x + 3) + 1$
So, $g(x) = \log_2(x + 3) + 1$

9. a) How is the graph of $y = 2\log_2(2x - 8)$ related to the graph of $y = \log_2 x$? Sketch both graphs on the same grid.

Compare $y = 2\log_2 2(x - 4)$ with $y - k = c\log_2 d(x - h)$:
$k = 0$, $c = 2$, $d = 2$, and $h = 4$
Write $y = 2\log_2(2x - 8)$ as $y = 2\log_2 2(x - 4)$. The graph of this function is the image of the graph of $y = \log_2 x$ after a vertical stretch by a factor of 2, a horizontal compression by a factor of $\frac{1}{2}$, then a translation of 4 units right.



Use the general transformation: (x, y) corresponds to $\left(\frac{x}{d} + h, cy + k\right)$

The point (x, y) on $y = \log_2 x$ corresponds to the point $\left(\frac{x}{2} + 4, 2y\right)$ on $y = 2\log_2 2(x - 4)$.

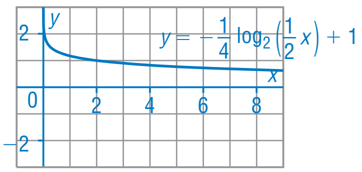
(x, y)	$\left(\frac{x}{2} + 4, 2y\right)$
(0.5, −1)	(4.25, −2)
(1, 0)	(4.5, 0)
(2, 1)	(5, 2)
(4, 2)	(6, 4)
(8, 3)	(8, 6)

b) Identify the intercepts and the equation of the asymptote of the graph of $y = 2\log_2(2x - 8)$, and the domain and range of the function. Use graphing technology to verify.

From the graph, there is no y-intercept.
From the table, the x-intercept is 4.5.
The equation of the asymptote is $x = 4$.
The domain of the function is $x > 4$.
The range of the function is $y \in \mathbb{R}$.

10. a) Graph $y = -\frac{1}{4}\log_2\left(\frac{1}{2}x\right) + 1$.

Compare $y - 1 = -\frac{1}{4}\log_2\left(\frac{1}{2}x\right)$
with $y - k = c\log_2 d(x-h)$:
$k = 1$, $c = -\frac{1}{4}$, $d = \frac{1}{2}$, and $h = 0$
Use the general transformation:
(x, y) corresponds to $\left(\frac{x}{d} + h, cy + k\right)$



The point (x, y) on $y = \log_2 x$ corresponds to the point $\left(2x, -\frac{1}{4}y + 1\right)$ on $y = -\frac{1}{4}\log_2\left(\frac{1}{2}x\right) + 1$.

(x, y)	$\left(2x, -\frac{1}{4}y + 1\right)$
(0.25, −2)	(0.5, 1.5)
(0.5, −1)	(1, 1.25)
(1, 0)	(2, 1)
(2, 1)	(4, 0.75)
(4, 2)	(8, 0.5)

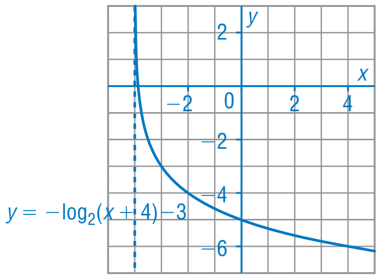
b) Identify the intercepts and the equation of the asymptote of the graph of $y = -\frac{1}{4}\log_2\left(\frac{1}{2}x\right) + 1$, and the domain and range of the function.

From the graph, there is no y-intercept.
Use the TABLE feature on a graphing calculator; the x-intercept is 32.
The equation of the asymptote is $x = 0$.
The domain of the function is $x > 0$.
The range of the function is $y \in \mathbb{R}$.

11. Graph each function below, then identify the intercepts and the equation of the asymptote of the graph, and the domain and range of the function.

a) $y = -\log_2(x + 4) - 3$

Compare $y + 3 = -\log_2(x + 4)$ with $y - k = c\log_2 d(x - h)$:
$k = -3$, $c = -1$, $d = 1$, and $h = -4$
Use the general transformation: (x, y) corresponds to $\left(\frac{x}{d} + h, cy + k\right)$
The point (x, y) on $y = \log_2 x$ corresponds to the point $(x - 4, -y - 3)$ on $y = -\log_2(x + 4) - 3$.



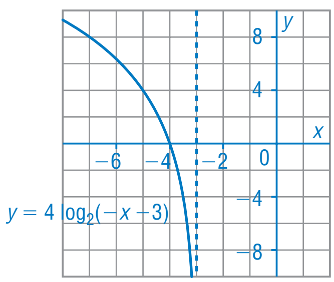
(x, y)	$(x - 4, -y - 3)$
$(0.25, -2)$	$(-3.75, -1)$
$(0.5, -1)$	$(-3.5, -2)$
$(1, 0)$	$(-3, -3)$
$(2, 1)$	$(-2, -4)$
$(4, 2)$	$(0, -5)$
$(8, 3)$	$(4, -6)$

From the graph, the x-intercept is approximately -3.9.
From the table, the y-intercept is -5.
The equation of the asymptote is $x = -4$.
The domain of the function is $x > -4$.
The range of the function is $y \in \mathbb{R}$.

b) $y = 4\log_2(-x - 3)$

Write $y = 4\log_2(-x - 3)$ as $y = 4\log_2[-(x + 3)]$.
Compare $y = 4\log_2[-(x + 3)]$ with $y - k = c\log_2 d(x - h)$:
$k = 0$, $c = 4$, $d = -1$, and $h = -3$
Use the general transformation: (x, y) corresponds to $\left(\frac{x}{d} + h, cy + k\right)$

The point (x, y) on $y = \log_2 x$ corresponds to the point $(-x - 3, 4y)$ on $y = 4\log_2(-x - 3)$.

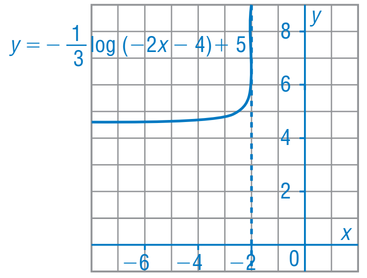


(x, y)	$(-x - 3, 4y)$
$(0.25, -2)$	$(-3.25, -8)$
$(0.5, -1)$	$(-3.5, -4)$
$(1, 0)$	$(-4, 0)$
$(2, 1)$	$(-5, 4)$
$(4, 2)$	$(-7, 8)$

From the table, the x-intercept is -4.
From the graph, there is no y-intercept.
The equation of the asymptote is $x = -3$.
The domain of the function is $x < -3$.
The range of the function is $y \in \mathbb{R}$.

C

12. Graph the function $y = -\frac{1}{3}\log_3(-2x - 4) + 5$, then identify the intercepts, the equation of the asymptote, and the domain and range of the function.



Write $y = -\frac{1}{3}\log_3(-2x - 4) + 5$ as $y - 5 = -\frac{1}{3}\log_3[-2(x + 2)]$.

Compare $y - 5 = -\frac{1}{3}\log_3[-2(x + 2)]$ with $y - k = c\log_3 d(x - h)$:

$k = 5, c = -\frac{1}{3}, d = -2$, and $h = -2$

Use the general transformation: (x, y) corresponds to $\left(\frac{x}{d} + h, cy + k\right)$

The point (x, y) on $y = \log_3 x$ corresponds to the point $\left(-\frac{1}{2}x - 2, -\frac{1}{3}y + 5\right)$ on $y = -\frac{1}{3}\log_3(-2x - 4) + 5$.

(x, y)	$\left(-\frac{1}{2}x - 2, -\frac{1}{3}y + 5\right)$
$\left(\frac{1}{9}, -2\right)$	$\left(-\frac{37}{18}, \frac{17}{3}\right)$
$\left(\frac{1}{3}, -1\right)$	$\left(-\frac{13}{6}, \frac{16}{3}\right)$
$(1, 0)$	$\left(-\frac{5}{2}, 5\right)$
$(3, 1)$	$\left(-\frac{7}{2}, \frac{14}{3}\right)$
$(9, 2)$	$\left(-\frac{13}{2}, \frac{13}{3}\right)$

To determine the x-intercept, solve the equation:

$$0 = -\frac{1}{3}\log_3(-2x - 4) + 5$$
$$-5 = -\frac{1}{3}\log_3(-2x - 4)$$
$$15 = \log_3(-2x - 4) \quad \text{Write in exponential form.}$$
$$-2x - 4 = 3^{15}$$
$$-2x = 14\,348\,911$$
$$x = -7\,174\,455.5$$

From the graph, there is no y-intercept.
The equation of the asymptote is $x = -2$.
The domain of the function is $x < -2$.
The range of the function is $y \in \mathbb{R}$.