

# CUMULATIVE REVIEW Chapters 1-5, pages 457–464

**1**

1. When  $bx^5 + x^4 - 5x^3 + x^2 + 2x - 4b$  is divided by  $x + 1$ , the remainder is  $-10$ . Determine the value of  $b$ .

$$\text{Let } P(x) = bx^5 + x^4 - 5x^3 + x^2 + 2x - 4b$$

Write  $x + 1$  as  $x - (-1)$

$$\begin{aligned} P(-1) &= b(-1)^5 + (-1)^4 - 5(-1)^3 + (-1)^2 + 2(-1) - 4b \\ &= -b + 1 + 5 + 1 - 2 - 4b \\ &= -5b + 5 \end{aligned}$$

The remainder is  $-10$ .

$$\text{So, } -5b + 5 = -10 \quad \text{Solve for } b.$$

$$-5b = -15$$

$$b = 3$$

The value of  $b$  is 3.

**2**

2. For the graph of each rational function below, determine without technology:

- i) the equations of any asymptotes and the coordinates of any hole
- ii) the domain of the function

Use graphing technology to verify the characteristics.

a)  $y = \frac{x^2 - 6x + 8}{x - 3}$

b)  $y = \frac{-3x + 12}{x^2 + 3}$

- i) The function is undefined when  $x - 3 = 0$ ; that is, when  $x = 3$ .

There are no common factors, so there is a vertical asymptote with equation  $x = 3$ .

The degree of the numerator is 1 more than that of the denominator, so there is an oblique asymptote.

Determine:

$$(x^2 - 6x + 8) \div (x - 3)$$

$$\begin{array}{r|rrr} 3 & 1 & -6 & 8 \\ & & 3 & -9 \\ \hline & 1 & -3 & -1 \end{array}$$

The quotient is  $x - 3$ , so the oblique asymptote has equation  $y = x - 3$ .

- ii) The domain is:  $x \neq 3$

- i) Since  $x^2 + 3$  is always positive, the function is defined for all real values of  $x$ .

The degree of the numerator is less than the degree of the denominator, so there is a horizontal asymptote with equation  $y = 0$ .

- ii) The domain is:  $x \in \mathbb{R}$

**3**

3. Describe how the graph of  $y = \sqrt{x}$  has been translated to create the graph of each function below. Use graphing technology to check.

a)  $y = \sqrt{x + 3}$

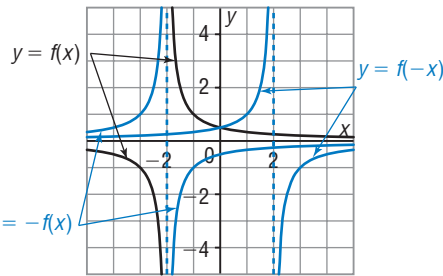
Write  $y = \sqrt{x + 3}$  as  $y = \sqrt{x - (-3)}$ , then compare to  $y - k = \sqrt{x - h}$ :  $h = -3$  and  $k = 0$

So, the graph of  $y = \sqrt{x + 3}$  is the graph of  $y = \sqrt{x}$  after a translation of 3 units left.

b)  $y = \sqrt{x + 1} + 1$

Write  $y = \sqrt{x + 1} + 1$  as  $y - 1 = \sqrt{x - (-1)}$ , then compare to  $y - k = \sqrt{x - h}$ :  $h = -1$  and  $k = 1$ . So, the graph of  $y = \sqrt{x + 1} + 1$  is the graph of  $y = \sqrt{x}$  after a translation of 1 unit left and 1 unit up.

4. Here is the graph of  $y = f(x)$ . On the same grid, sketch and label the graph of each function.



a)  $y = f(-x)$

The graph of  $y = f(-x)$  is the image of the graph of  $y = f(x)$  after a reflection in the  $y$ -axis.

Choose points on  $y = f(x)$ , then reflect them in the  $y$ -axis. The line  $x = -2$  is an asymptote and its reflection has equation  $x = 2$ . Join the points with 2 smooth curves to form the graph of  $y = f(-x)$ .

b)  $y = -f(x)$

The graph of  $y = -f(x)$  is the image of the graph of  $y = f(x)$  after a reflection in the  $x$ -axis. Choose points on  $y = f(x)$ , then reflect them in the  $x$ -axis. The line  $x = -2$  is not changed by the reflection. Join the points with 2 smooth curves to form the graph of  $y = -f(x)$ .

5. Graph  $y = \sqrt{x}$ , then apply transformations to sketch the graph of  $y - 2 = -\frac{1}{2}\sqrt{3x}$ . What are the domain and range of this function?

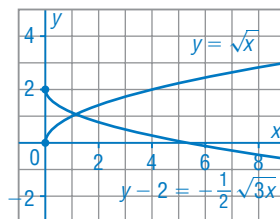
Compare  $y - 2 = -\frac{1}{2}\sqrt{3x}$  to

$y - k = a\sqrt{b(x - h)}$ :

$k = 2, a = -\frac{1}{2}, b = 3, \text{ and } h = 0$

$(x, y)$  on  $y = \sqrt{x}$  corresponds to

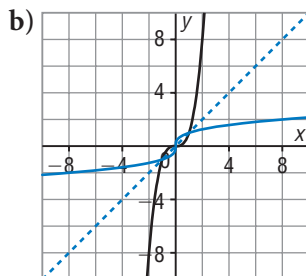
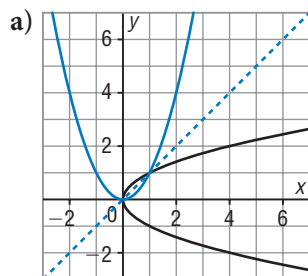
$(\frac{x}{3}, -\frac{1}{2}y + 2)$  on  $y - 2 = -\frac{1}{2}\sqrt{3x}$ .



Point on $y = \sqrt{x}$	Point on $y - 2 = -\frac{1}{2}\sqrt{3x}$
$(0, 0)$	$(0, 2)$
$(1, 1)$	$(\frac{1}{3}, \frac{3}{2})$
$(9, 3)$	$(3, \frac{1}{2})$

The domain is:  $x \geq 0$ ; the range is:  $y \leq 2$

6. For each graph of a relation below, sketch the graph of its inverse. Is the inverse a function? How do you know?



Reflect each graph in the line  $y = x$ .

The inverse is a function because its graph passes the vertical line test.

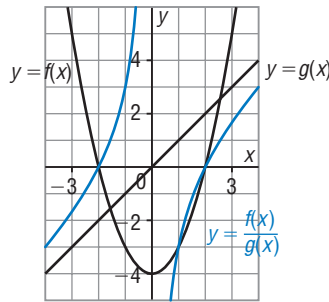
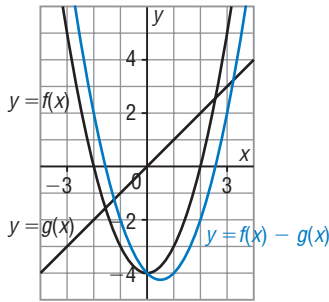
The inverse is a function because its graph passes the vertical line test.

4

7. Use the graphs of  $y = f(x)$  and  $y = g(x)$  to sketch the graph of each function below, then identify its domain and range.

a)  $y = f(x) - g(x)$

b)  $y = \frac{f(x)}{g(x)}$



From the graphs:

$x$	$f(x)$	$g(x)$	$f(x) - g(x)$	$\frac{f(x)}{g(x)}$
-3	5	-3	8	$-1.\bar{6}$
-2	0	-2	2	0
-1	-3	-1	-2	3
0	-4	0	-4	undefined
1	-3	1	-4	-3
2	0	2	-2	0
3	5	3	2	$1.\bar{6}$

Plot points with coordinates  $(x, f(x) - g(x))$ , which fit on the grid. Join the points with a smooth curve. From the graph, the domain is:  $x \in \mathbb{R}$ ; and the range appears to be  $y \geq -4.25$

Plot points with coordinates  $(x, \frac{f(x)}{g(x)})$ . Since  $x \neq 0$ , the  $y$ -axis is an asymptote, so join the points with 2 smooth curves. From the graph, the domain is:  $x \neq 0$ ; and the range is:  $y \in \mathbb{R}$

8. Given that  $f(x) = 2 + x^2$ ,  $g(x) = -3x + 5$ , and  $h(x) = \sqrt{2 - 4x}$ , write an explicit equation for  $k(x)$  then state its domain.

a)  $k(x) = f(x) + g(x) + h(x)$       b)  $k(x) = f(x) \cdot g(x) - h(x)$

$$k(x) = 2 + x^2 - 3x + 5 + \sqrt{2 - 4x}$$

$$k(x) = x^2 - 3x + 7 + \sqrt{2 - 4x}$$

For the domain,  $2 - 4x \geq 0$ ,

$$\text{so } x \leq 0.5$$

$$k(x) = (2 + x^2)(-3x + 5) - \sqrt{2 - 4x}$$

$$k(x) = -3x^3 + 5x^2 - 6x + 10 - \sqrt{2 - 4x}$$

For the domain,  $2 - 4x \geq 0$ ,

$$\text{so } x \leq 0.5$$

9. Use the functions  $f(x) = x^2 - x$ ,  $g(x) = \frac{1}{3 - x}$ , and  $h(x) = |x + 1|$ .

a) Determine each value.

i)  $f(g(4))$

$$g(4) = \frac{1}{3 - 4}, \text{ or } -1$$

$$f(g(4)) = (-1)^2 - (-1) = 2$$

ii)  $g(h(-3))$

$$h(-3) = |-3 + 1|, \text{ or } 2$$

$$g(h(-3)) = \frac{1}{3 - 2} = 1$$

b) Write an explicit equation for each composite function, then state its domain.

i)  $h(g(x))$

$$h(g(x)) = \left| \frac{1}{3 - x} + 1 \right|$$

$$h(g(x)) = \left| \frac{4 - x}{3 - x} \right|$$

For the domain,  $3 - x \neq 0$ ,

$$\text{so } x \neq 3$$

ii)  $f(f(x))$

$$f(f(x)) = (x^2 - x)^2 - (x^2 - x)$$

$$f(f(x)) = x^4 - 2x^3 + x^2 - x^2 + x$$

$$f(f(x)) = x^4 - 2x^3 + x$$

This is a quartic function; its

domain is:  $x \in \mathbb{R}$

10. For the function  $y = 3x^2 - 12x + 5$ , determine possible functions  $f$  and  $g$  so that  $y = f(g(x))$ .

Sample response:

Complete the square.

$$y = 3(x^2 - 4x + 4 - 4) + 5$$

$$y = 3(x^2 - 4x + 4) - 7$$

$$y = 3(x - 2)^2 - 7$$

$$\text{Let } f(g(x)) = 3(x - 2)^2 - 7$$

$$\text{Then } g(x) = x - 2 \text{ and } f(x) = 3x^2 - 7$$

## 5

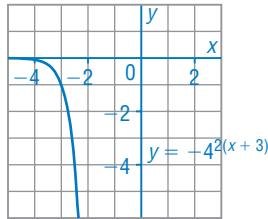
11. Use transformations to graph  $y = -4^{2(x+3)}$ . Describe the transformations, then state the characteristics of the graph; include the intercepts, equation of the asymptote, domain, and range.

Compare  $y = -4^{2(x+3)}$  with  $y - k = c4^{d(x-h)}$ :

$k = 0$ ,  $c = -1$ ,  $d = 2$ , and  $h = -3$

The graph of  $y = -4^{2(x+3)}$  is the image of the graph of  $y = 4^x$  after a horizontal compression by a factor of  $\frac{1}{2}$ , a reflection in the  $x$ -axis, then a translation of 3 units left.

A point  $(x, y)$  on  $y = 4^x$  corresponds to the point  $(\frac{x}{2} - 3, -y)$  on  $y = -4^{2(x+3)}$ .



$(x, y)$	$(\frac{x}{2} - 3, -y)$
$(-2, 0.0625)$	$(-4, -0.0625)$
$(-1, 0.25)$	$(-3.5, -0.25)$
$(0, 1)$	$(-3, -1)$
$(1, 4)$	$(-2.5, -4)$
$(2, 16)$	$(-2, -16)$

The function is decreasing. There is no  $x$ -intercept.

For the  $y$ -intercept, substitute  $x = 0$  in  $y = -4^{2(x+3)}$ .

$$y = -4^{2(0+3)}$$

$$y = -4^6 \text{ or } -4096$$

The horizontal asymptote has equation  $y = 0$ .

The domain is:  $x \in \mathbb{R}$ ; and the range is:  $y < 0$

12. A cup of coffee contains 150 mg of caffeine. In a healthy adult, the mass,  $m$  grams, that remains after  $t$  hours is modelled by the function  $m = 150(0.5)^{0.2t}$ . Use technology.
- a) To the nearest half hour, how long will it take until only 10 mg of caffeine remain?

Use  $m = 150(0.5)^{0.2t}$       Substitute:  $m = 10$

$$10 = 150(0.5)^{0.2t}$$

Graph the related function:  $y = 150(0.5)^{0.2t} - 10$

Determine the approximate zero of the function: 19.534453

It will take approximately 19.5 h until only 10 mg of caffeine remain.

- b) To the nearest tenth of a milligram, how much caffeine remains after 24 h?

Use  $m = 150(0.5)^{0.2t}$       Substitute:  $t = 24$

$$m = 150(0.5)^{0.2(24)}$$

$$m = 5.3845\dots$$

After 24 h, approximately 5.4 mg of caffeine remain.

13. Solve each equation.

a)  $9^{2x+1} = \frac{\sqrt[4]{27}}{81}$

$$3^{2(2x+1)} = \frac{3^{\frac{3}{4}}}{3^4}$$

$$3^{2(2x+1)} = 3^{-\frac{13}{4}}$$

$$2(2x + 1) = -\frac{13}{4}$$

$$2x + 1 = -\frac{13}{8}$$

$$2x = -\frac{21}{8}$$

$$x = -\frac{21}{16}$$

Use a calculator to verify.

b)  $\log_3(x + 5) + \log_3(x - 1)$   
 $= 2 + \log_3(x + 1)$

$$x > -5, x > 1, \text{ and } x > -1, \text{ so } x > 1$$

$$\log_3(x + 5)(x - 1) = \log_3 9 + \log_3(x + 1)$$

$$(x + 5)(x - 1) = 9(x + 1)$$

$$x^2 + 4x - 5 = 9x + 9$$

$$x^2 - 5x - 14 = 0$$

$$(x + 2)(x - 7) = 0$$

$$x = -2 \text{ or } x = 7$$

$$x = -2 \text{ is extraneous, because } x > 1$$

Verify  $x = 7$ :

$$\text{L.S.} = \log_3 12 + \log_3 6, \text{ or } \log_3 72$$

$$\text{R.S.} = \log_3 9 + \log_3 8, \text{ or } \log_3 72$$

The solution is verified.

14. Write each expression in terms of  $\log a$ ,  $\log b$ , and  $\log c$ .

a)  $\log\left(\frac{a^3\sqrt{b}}{c}\right)$

$$= \log a^3 b^{\frac{1}{2}} - \log c$$

$$= \log a^3 + \log b^{\frac{1}{2}} - \log c$$

$$= 3 \log a + \frac{1}{2} \log b - \log c$$

b)  $\log\left(\frac{a^{\frac{2}{3}}}{b^4\sqrt[4]{c^3}}\right)$

$$= \log a^{\frac{2}{3}} - \log b^4 \sqrt[4]{c^3}$$

$$= \log a^{\frac{2}{3}} - (\log b + \log \sqrt[4]{c^3})$$

$$= \frac{2}{3} \log a - \log b - \log c^{\frac{3}{4}}$$

$$= \frac{2}{3} \log a - \log b - \frac{3}{4} \log c$$

15. Write each expression as a single logarithm.

a)  $4 \log 3 - 2 \log 6 + 2 \log 2$       b)  $2 \log_7 x - \log_7 y - \frac{1}{4} \log_7 z$

$$= \log 3^4 - \log 6^2 + \log 2^2$$

$$= \log 81 - \log 36 + \log 4$$

$$= \log\left(\frac{81 \cdot 4}{36}\right)$$

$$= \log 9$$

$$= \log_7 x^2 - \log_7 y - \log_7 z^{\frac{1}{4}}$$

$$= \log_7\left(\frac{x^2}{yz^{\frac{1}{4}}}\right)$$

- 16.** On March 11, 2011, an earthquake off the coast of Japan had a magnitude of 9.0 and it caused a tsunami. To the nearest tenth, determine the magnitude of an earthquake that is one-half as intense as this earthquake.

Use:  $M = \log \left( \frac{I}{S} \right)$       Substitute:  $M = 9.0$

$$9.0 = \log \left( \frac{I}{S} \right)$$

$$\frac{I}{S} = 10^{9.0}$$

$$I = 10^{9.0}S$$

An earthquake that is one-half as intense has intensity:  $I = 0.5(10^{9.0}S)$

Work backward.

$$\frac{I}{S} = 0.5(10^{9.0}) \quad \text{Take the common logarithm of each side.}$$

$$\log \left( \frac{I}{S} \right) = \log [0.5(10^{9.0})]$$

$$M = 8.6989 \dots$$

The magnitude is approximately 8.7.