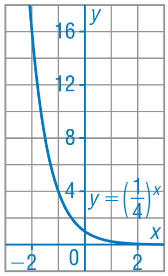
REVIEW, pages 444–452

5.1

1. Complete the table of values, then graph $y = \left(\frac{1}{4}\right)^x$.

x	−2	−1	0	1	2
y	16	4	1	0.25	0.0625

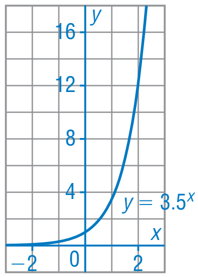


5.2

2. a) Graph $y = 3.5^x$ for $-2 \leq x \leq 2$.

Make a table of values.
Write the coordinates to the nearest hundredth.

x	−2	−1	0	1	2
y	0.08	0.29	1	3.5	12.25



b) Determine:

i) whether the function is increasing or decreasing

The function is increasing.

ii) the intercepts

There is no *x*-intercept; the *y*-intercept is 1.

iii) the equation of the asymptote

The asymptote has equation $y = 0$.

iv) the domain of the function

The domain is $x \in \mathbb{R}$.

v) the range of the function

The range is $y > 0$.

3. Use technology to graph each function below. For each graph:

i) identify the intercepts

ii) identify the equation of the asymptote and state why it is significant

a) $y = 0.8^x$

i) There is no x-intercept. The y-intercept is 1.

ii) The equation of the asymptote is $y = 0$. This is the line that the graph approaches as x increases.

b) $y = 2.75^x$

i) There is no x-intercept. The y-intercept is 1.

ii) The equation of the asymptote is $y = 0$. This is the line that the graph approaches as x decreases.

4. a) Sketch the graph of $y = -\frac{1}{2}(3^{2x}) - 1$.

Write the function as: $y + 1 = -\frac{1}{2}(3^{2x})$

Compare $y + 1 = -\frac{1}{2}(3^{2x})$ with $y - k = c3^{d(x - h)}$:

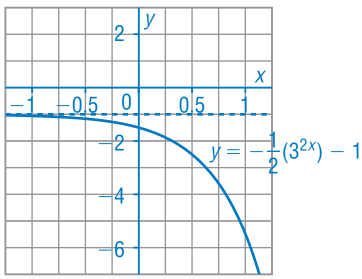
$k = -1$, $c = -\frac{1}{2}$, $d = 2$, and $h = 0$

Use the general transformation:

(x, y) corresponds to $\left(\frac{x}{d} + h, cy + k\right)$

The point (x, y) on $y = 3^x$ corresponds to the point $\left(\frac{x}{2}, -\frac{1}{2}y - 1\right)$ on $y + 1 = -\frac{1}{2}(3^{2x})$.

Choose points (x, y) on $y = 3^x$.



(x, y)	$\left(\frac{x}{2}, -\frac{1}{2}y - 1\right)$
$\left(-2, \frac{1}{9}\right)$	$\left(-1, -\frac{19}{18}\right)$
$\left(-1, \frac{1}{3}\right)$	$\left(-\frac{1}{2}, -\frac{7}{6}\right)$
$(0, 1)$	$\left(0, -\frac{3}{2}\right)$
$(1, 3)$	$\left(\frac{1}{2}, -\frac{5}{2}\right)$
$(2, 9)$	$\left(1, -\frac{11}{2}\right)$

b) From the graph, identify:

i) whether the function is increasing or decreasing

The function is decreasing.

ii) the intercepts

There is no x-intercept. From the table, the y-intercept is -1.5.

iii) the equation of the asymptote

The asymptote has equation $y = -1$.

iv) the domain of the function

The domain of the function is $x \in \mathbb{R}$.

v) the range of the function

The range of the function is $y < -1$.

5.3

5. Solve each equation.

a) $4^x = 128$

$$2^{2x} = 2^7$$
$$2x = 7$$
$$x = 3.5$$

b) $27^{x+1} = 81^{x-2}$

$$3^{3(x+1)} = 3^{4(x-2)}$$
$$3x + 3 = 4x - 8$$
$$x = 11$$

c) $9^x = 27\sqrt[4]{3}$

$$3^{2x} = (3^3)\left(3^{\frac{1}{4}}\right)$$
$$2x = 3.25$$
$$x = 1.625$$

d) $\dfrac{\sqrt[3]{2}}{8} = 4^x$

$$\left(2^{\frac{1}{3}}\right)(2^{-3}) = 2^{2x}$$
$$-\frac{8}{3} = 2x$$
$$x = -\frac{4}{3}$$

6. Solve the equation $1.04^{2x} = 2$. Give the solution to the nearest tenth.

Use technology to graph $y = 1.04^{2x}$ and $y = 2$.
Determine the approximate x-coordinate of the point of intersection:
8.8364938
The solution is: $x \doteq 8.8$

7. A new combine, used for harvesting wheat, costs \$370 000. Its value depreciates by 10% each year. The value of the combine, v thousands of dollars, after t years can be modelled by this function: $v = 370(0.9)^t$

a) What is the value of the combine when it is 5 years old? Give the answer to the nearest thousand dollars.

Use technology to graph $y = 370(0.9)^x$ for $0 < x < 15$.
Press: TRACE 5 ENTER to display:
X = 5 Y = 218.4813
After 5 years, the value of the combine is approximately \$218 000.

b) When will the combine be worth \$100 000? Give the answer to the nearest half year.

Graph $y = 100$ on the same screen as $y = 370(0.9)^x$.
Use 5: intersect from the CALC menu to display:
X = 12.417677 Y = 100
The combine will be worth \$100 000 after approximately 12.5 years.

8. A principal of \$2500 is invested at 3% annual interest, compounded semi-annually. To the nearest year, how long will it be until the amount is \$3000?

Use the formula:
$A = A_0\left(1 + \frac{i}{n}\right)^{nt}$ **Substitute: $A = 3000$, $A_0 = 2500$, $i = 0.03$, $n = 2$**

$3000 = 2500(1.015)^{2t}$
Use technology to graph $y = 2500(1.015)^{2x}$ and $y = 3000$ for $0 < x < 10$.
Use 5: intersect from the CALC menu to display:
X = 6.1228525 Y = 3000
After approximately 6 years, the amount will be \$3000.

5.4

9. a) Write each logarithmic expression as an exponential expression.

i) $\log_3 729 = 6$

The base is 3.
The exponent is 6.
So, $729 = 3^6$

ii) $\log_4 2\sqrt{2} = \frac{3}{4}$

The base is 4.
The exponent is $\frac{3}{4}$.
So, $2\sqrt{2} = 4^{\frac{3}{4}}$

b) Write each exponential expression as a logarithmic expression.

i) $4^5 = 1024$

The base is 4.
The logarithm is 5.
So, $5 = \log_4 1024$

ii) $5^{-4} = \frac{1}{625}$

The base is 5.
The logarithm is −4.
So, $-4 = \log_5\left(\frac{1}{625}\right)$

10. For each logarithm below, determine its exact value or use benchmarks to determine its approximate value to the nearest tenth.

a) $\log_7 343$

$= \log_7(7^3)$
$= 3$

b) $\log_8 100$

Identify powers of 8 close to 100.
$8^2 = 64$ and $8^3 = 512$
So, $2 < \log_8 100 < 3$
An estimate is: $\log_8 100 \doteq 2.2$
Check.
$8^{2.2} \doteq 97.00586026$
$8^{2.3} \doteq 119.4282229$
So, $\log_8 100 \doteq 2.2$

c) $\log_2 20$

Identify powers of 2 close to 20.
$2^4 = 16$ and $2^5 = 32$
So, $4 < \log_2 20 < 5$
An estimate is: $\log_2 20 \doteq 4.3$
Check.
$2^{4.3} \doteq 19.69831061$
$2^{4.4} \doteq 21.11212657$
So, $\log_2 20 \doteq 4.3$

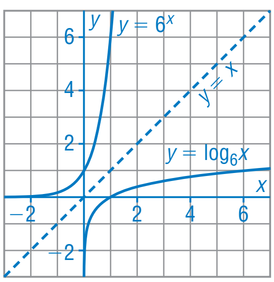
d) $\log_4\left(\frac{1}{32}\right)$

$= \log_4(2^{-5})$
$= \log_4(4^{\frac{1}{2}})^{-5}$
$= \log_4(4^{-\frac{5}{2}})$
$= -\frac{5}{2}$

11. a) Graph $y = \log_6 x$.

Determine values for $y = 6^x$, then interchange the coordinates for the table of values for $y = \log_6 x$.

x	$y = \log_6 x$
$\frac{1}{6}$	−1
1	0
6	1



b) Identify the intercepts and the equation of the asymptote of the graph, and the domain and range of the function.

There is no y-intercept. The x-intercept is 1.
The asymptote has equation $x = 0$.
The domain is $x > 0$. The range is $y \in \mathbb{R}$.

c) How could you use the graph of $y = \log_6 x$ to graph $y = 6^x$? Use your strategy to graph $y = 6^x$ on the grid in part a.

I reflect points on the graph of $y = \log_6 x$ in the line $y = x$, then join the points for the graph of $y = 6^x$.

5.5

12. Write each expression as a single logarithm.

a) $3 \log x + \frac{1}{2} \log y - 2 \log z$

$$= \log x^3 + \log y^{\frac{1}{2}} - \log z^2$$
$$= \log\left(\frac{x^3 y^{\frac{1}{2}}}{z^2}\right)$$

b) $4 + \log_2 3$

$$= \log_2 16 + \log_2 3$$
$$= \log_2 48$$

13. Evaluate: $2 \log_4 6 - \log_4 18 + \log_4 8$

$$= \log_4 6^2 + \log_4 8 - \log_4 18$$
$$= \log_4\left(\frac{36 \cdot 8}{18}\right)$$
$$= \log_4 16$$
$$= \log_4 4^2$$
$$= 2$$

5.6

14. Approximate the value of each logarithm, to the nearest thousandth.

a) $\log_5 600$

$$\log_5 600 = \frac{\log 600}{\log 5}$$
$$= 3.9746\ldots$$
$$\doteq 3.975$$

b) $\log_3 0.1$

$$\log_3 0.1 = \frac{\log 0.1}{\log 3}$$
$$= -2.0959\ldots$$
$$\doteq -2.096$$

15. Use technology to graph $y = \log_9 x$. Identify the intercepts and the equation of the asymptote of the graph, and the domain and range of the function.

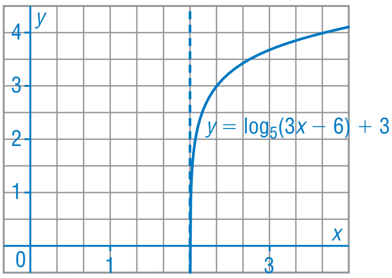
Graph: $y = \frac{\log x}{\log 9}$

Use the zero feature from the CALC menu; the x-intercept is 1. There is no y-intercept.

The equation of the asymptote is $x = 0$.

The domain of the function is $x > 0$. The range of the function is $y \in \mathbb{R}$.

16. a) Sketch the graph of $y = \log_5(3x - 6) + 3$.



Write $y = \log_5(3x - 6) + 3$ as $y - 3 = \log_5 3(x - 2)$.

Compare $y - 3 = \log_5 3(x - 2)$ with $y - k = c \log_5 d(x - h)$:

$k = 3$, $c = 1$, $d = 3$, and $h = 2$

Use the general transformation: (x, y) corresponds to $\left(\frac{x}{d} + h, cy + k\right)$

The point (x, y) on $y = \log_5 x$ corresponds to the point $\left(\frac{x}{3} + 2, y + 3\right)$ on $y = \log_5(3x - 6) + 3$.

(x, y)	$\left(\frac{x}{3} + 2, y + 3\right)$
$\left(\frac{1}{25}, -2\right)$	$\left(\frac{151}{75}, 1\right)$
$\left(\frac{1}{5}, -1\right)$	$\left(\frac{31}{15}, 2\right)$
$(1, 0)$	$\left(\frac{7}{3}, 3\right)$
$(5, 1)$	$\left(\frac{11}{3}, 4\right)$

b) Identify the intercepts and the equation of the asymptote of the graph of $y = \log_5(3x - 6) + 3$, and the domain and range of this function.

There is no y-intercept.
For the x-intercept, substitute $y = 0$ in $y = \log_5(3x - 6) + 3$, then solve for x.

$$0 = \log_5(3x - 6) + 3$$
$$\log_5(3x - 6) = -3$$
$$3x - 6 = 5^{-3}$$
$$3x = 6 + \frac{1}{125}$$
$$3x = \frac{751}{125}$$
$$x = \frac{751}{375}$$

The equation of the asymptote is $x = 2$.
The domain of the function is $x > 2$.
The range of the function is $y \in \mathbb{R}$.

5.7

17. Solve, then verify each logarithmic equation.

a) $3 = \log_2(x + 5) + \log_2(x + 7)$

$x > -5$ and $x > -7$; so $x > -5$

$$3 = \log_2(x + 5)(x + 7)$$
$$2^3 = (x + 5)(x + 7)$$
$$x^2 + 12x + 27 = 0$$
$$(x + 9)(x + 3) = 0$$
$x = -9$ or $x = -3$
$x = -9$ is extraneous.
Verify: $x = -3$

$$\text{R.S.} = \log_2 2 + \log_2 4 = 1 + 2 = 3 = \text{L.S.}$$

The solution is verified.

b) $\log x + \log (x + 1) = \log (7x - 8)$

$x > 0$, $x > -1$, $x > \frac{8}{7}$; so $x > \frac{8}{7}$

$$\log x(x + 1) = \log (7x - 8)$$
$$x(x + 1) = 7x - 8$$
$$x^2 - 6x + 8 = 0$$
$$(x - 2)(x - 4) = 0$$
$x = 2$ or $x = 4$
Verify $x = 2$:
L.S. = log 6 R.S. = log 6
The solution is verified.
Verify $x = 4$:
L.S. = log 20 R.S. = log 20
The solution is verified.

18. Solve each equation algebraically. Give the solution to the nearest hundredth.

a) $5(3^x) = 60$

$$3^x = 12$$
$$\log_3 3^x = \log_3 12$$
$$x = \frac{\log 12}{\log 3}$$
$$x \doteq 2.26$$

b) $3^{x+4} = 5^{x+1}$

$$\log 3^{x+4} = \log 5^{x+1}$$
$$(x + 4)\log 3 = (x + 1)\log 5$$
$$x \log 3 + 4 \log 3 = x \log 5 + \log 5$$
$$x(\log 3 - \log 5) = \log 5 - 4 \log 3$$
$$x = \frac{\log 5 - 4 \log 3}{\log 3 - \log 5}$$
$$x \doteq 5.45$$

5.8

19. The pH of a solution can be described by the equation $\text{pH} = -\log [H^+]$, where $[H^+]$ is the hydrogen-ion concentration in moles/litre.

a) Determine the hydrogen-ion concentration in pure water with a pH of 7.

Substitute pH = 7 in the equation: $\text{pH} = -\log [H^+]$
$7 = -\log [H^+]$ Write in exponential form.
$[H^+] = 10^{-7}$
The hydrogen-ion concentration of pure water is 10^{-7} moles/litre.

b) How are the hydrogen-ion concentrations of these liquids related: black coffee with a pH of 5 and pure water?

For the hydrogen-ion concentration of black coffee, substitute pH = 5 in the equation: $\text{pH} = -\log [H^+]$
$5 = -\log [H^+]$ Write in exponential form.
$[H^+] = 10^{-5}$
The hydrogen-ion concentration of black coffee is 10^{-5} moles/litre.

$$\frac{10^{-5}}{10^{-7}} = 10^2, \text{ or } 100$$

So, black coffee has 100 times as many hydrogen ions per litre as pure water.