Lesson 6.1 Exercises, pages 474–480

A

3. Use technology to determine the value of each trigonometric ratio to the nearest thousandth.

a) sin 415°

$\doteq$ **0.819**

b) cos (−65°)

$\doteq$ **0.423**

c) cot 72°

$= \dfrac{1}{\tan 72°}$

$\doteq$ **0.325**

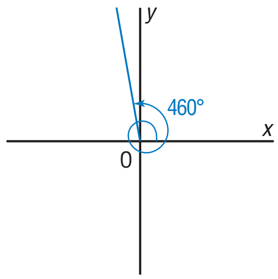
d) csc 285°

$= \dfrac{1}{\sin 285°}$

$\doteq$ **−1.035**

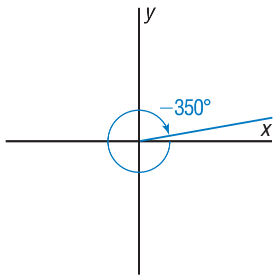
4. Sketch each angle in standard position, then identify the reference angle.

a) 460°



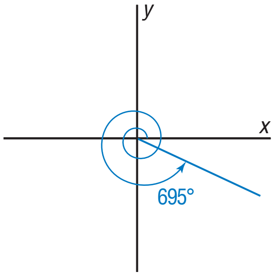
A coterminal angle is:
460° − 360° = 100°
The reference angle is:
180° − 100° = 80°

b) −350°



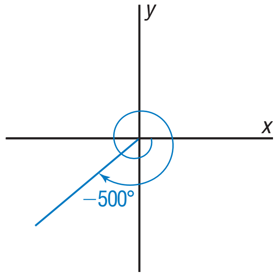
A coterminal angle is:
360° − 350° = 10°
This is also the reference angle.

c) 695°



A coterminal angle is:
$-720° + 695° = -25°$
The reference angle is: 25°

d) $-500°$



A coterminal angle is:
$360° - 500° = -140°$
The reference angle is:
$180° - 140° = 40°$

5. For each angle in standard position below:

i) Determine the measures of angles that are coterminal with the angle in the given domain.

ii) Write an expression for the measures of all the angles that are coterminal with the angle in standard position.

a) 75°;
for $-500° \leq \theta \leq 500°$

i) Between 500° and 0°, the coterminal angles are:
75°; and $75° + 360° = 435°$
Between 0° and $-500°$, the coterminal angle is:
$75° - 360° = -285°$

ii) The measures of all the angles coterminal with 75° can be represented by the expression:
$75° + k360°, k \in \mathbb{Z}$

b) $-105°$;
for $-600° \leq \theta \leq 600°$

i) Between 600° and 0°, the coterminal angle is:
$-105° + 360° = 255°$
Between 0° and $-600°$, the coterminal angles are:
$-105°$; and
$-105° - 360° = -465°$

ii) The measures of all the angles coterminal with $-105°$ can be represented by the expression:
$-105° + k360°, k \in \mathbb{Z}$

c) 215°;
for $-700° \leq \theta \leq 700°$

i) Between 700° and 0°, the coterminal angles are:
215°; and
$215° + 360° = 575°$
Between 0° and $-700°$, the coterminal angles are:
$215° - 360° = -145°$; and
$215° - 2(360°) = -505°$

ii) The measures of all the angles coterminal with 215° can be represented by the expression: $215° + k360°$,
$k \in \mathbb{Z}$

d) $-290°$;
for $-800° \leq \theta \leq 800°$

i) Between 800° and 0°, the coterminal angles are:
$-290° + 360° = 70°$;
$-290° + 2(360°) = 430°$; and
$-290° + 3(360°) = 790°$
Between 0° and $-800°$, the coterminal angles are:
$-290°$; and
$-290° - 360° = -650°$

ii) The measures of all the angles coterminal with $-290°$ can be represented by the expression:
$-290° + k360°, k \in \mathbb{Z}$

B

6. Use exact values to complete this table.

You will return to this table when you complete Lesson 6.3 Exercises.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
0°, 0	0	1	0	–	1	–
30°, $\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$	$\sqrt{3}$
45°, $\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°, $\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2	$\frac{1}{\sqrt{3}}$
90°, $\frac{\pi}{2}$	1	0	–	1	–	0
120°, $\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$\frac{2}{\sqrt{3}}$	-2	$-\frac{1}{\sqrt{3}}$
135°, $\frac{3\pi}{4}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	-1	$\sqrt{2}$	$-\sqrt{2}$	-1
150°, $\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$	2	$-\frac{2}{\sqrt{3}}$	$-\sqrt{3}$
180°, π	0	-1	0	–	-1	–
210°, $\frac{7\pi}{6}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	-2	$-\frac{2}{\sqrt{3}}$	$\sqrt{3}$
225°, $\frac{5\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	1	$-\sqrt{2}$	$-\sqrt{2}$	1
240°, $\frac{4\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$	$-\frac{2}{\sqrt{3}}$	-2	$\frac{1}{\sqrt{3}}$
270°, $\frac{3\pi}{2}$	-1	0	–	-1	–	0
300°, $\frac{5\pi}{3}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$	$-\frac{2}{\sqrt{3}}$	2	$-\frac{1}{\sqrt{3}}$
315°, $\frac{7\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	-1	$-\sqrt{2}$	$\sqrt{2}$	-1
330°, $\frac{11\pi}{6}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$	-2	$\frac{2}{\sqrt{3}}$	$-\sqrt{3}$
360°, 2π	0	1	0	–	1	–

7. Determine the exact values of the 6 trigonometric ratios for each angle.

a) 480°

A coterminal angle is: $480° - 360° = 120°$
From the completed table in question 6:

$\sin 480° = \sin 120° = \frac{\sqrt{3}}{2}$ $\quad \cos 480° = \cos 120° = -\frac{1}{2}$ $\quad \tan 480° = \tan 120° = -\sqrt{3}$

$\csc 480° = \frac{1}{\sin 120°} = \frac{2}{\sqrt{3}}$ $\quad \sec 480° = \frac{1}{\cos 120°} = -2$ $\quad \cot 480° = \frac{1}{\tan 120°} = -\frac{1}{\sqrt{3}}$

b) −855°

A coterminal angle is: $1080° - 855° = 225°$
From the completed table in question 6:

$\sin(-855°) = \sin 225° = -\frac{1}{\sqrt{2}}$ $\quad \cos(-855°) = \cos 225° = -\frac{1}{\sqrt{2}}$ $\quad \tan(-855°) = \tan 225° = 1$

$\csc(-855°) = \frac{1}{\sin 225°} = -\sqrt{2}$ $\quad \sec(-855°) = \frac{1}{\cos 225°} = -\sqrt{2}$ $\quad \cot(-855°) = \frac{1}{\tan 225°} = 1$

8. For each point $P(x, y)$ on the terminal arm of an angle θ in standard position, determine the exact values of the six trigonometric ratios for θ.

a) P(2, 1)

Let the distance between the origin and P be r.
Use: $x^2 + y^2 = r^2$
Substitute: $x = 2, y = 1$
$2^2 + 1^2 = r^2$
$r = \sqrt{5}$
The terminal arm lies in Quadrant 1.

$\sin\theta = \frac{1}{\sqrt{5}}$ $\quad \csc\theta = \sqrt{5}$

$\cos\theta = \frac{2}{\sqrt{5}}$ $\quad \sec\theta = \frac{\sqrt{5}}{2}$

$\tan\theta = \frac{1}{2}$ $\quad \cot\theta = 2$

b) P(3, −4)

Let the distance between the origin and P be r.
Use: $x^2 + y^2 = r^2$
Substitute: $x = 3, y = -4$
$3^2 + (-4)^2 = r^2$
$r = 5$
The terminal arm lies in Quadrant 4.

$\sin\theta = -\frac{4}{5}$ $\quad \csc\theta = -\frac{5}{4}$

$\cos\theta = \frac{3}{5}$ $\quad \sec\theta = \frac{5}{3}$

$\tan\theta = -\frac{4}{3}$ $\quad \cot\theta = -\frac{3}{4}$

9. For each point P(x, y) on the terminal arm of an angle θ in standard position, determine possible measures of θ in the given domain. Give the answers to the nearest degree.

a) P(−4, 2); for $-360° \leq \theta \leq 0°$

The terminal arm of angle θ lies in Quadrant 2.
The reference angle is:
$\tan^{-1}\left(\frac{2}{4}\right) \doteq 27°$
So, $\theta \doteq 180° - 27°$, or 153°
The angle between −360° and 0° that is coterminal with 153° is:
$-360° + 153° = -207°$
Possible value of θ is: approximately −207°

b) P(−4, −8); for $-360° \leq \theta \leq 360°$

The terminal arm of angle θ lies in Quadrant 3.
The reference angle is:
$\tan^{-1}\left(\frac{8}{4}\right) \doteq 63°$
So, $\theta \doteq 180° + 63°$, or 243°
The angle between −360° and 0° that is coterminal with 243° is:
$-360° + 243° = -117°$
Possible values of θ are approximately: 243° and −117°

10. For each trigonometric ratio, determine the exact values of the other 5 trigonometric ratios for θ in the given domain.

a) $\cos\theta = \frac{1}{\sqrt{2}}$; for $0° \leq \theta \leq 180°$

b) $\cot\theta = -\sqrt{3}$; for $90° \leq \theta \leq 270°$

Use the completed table in question 6.

a) **From the given domain, $\theta = 45°$**
Then: $\cos\theta = \frac{1}{\sqrt{2}}$, $\sec\theta = \sqrt{2}$,
$\sin\theta = \frac{1}{\sqrt{2}}$, $\csc\theta = \sqrt{2}$,
$\tan\theta = 1$, and $\cot\theta = 1$

b) **$\cot\theta = -\sqrt{3}$ for $\theta = 150°$ or $\theta = 330°$**
From the given domain, $\theta = 150°$
Then: $\cot\theta = -\sqrt{3}$, $\tan\theta = -\frac{1}{\sqrt{3}}$,
$\sin\theta = \frac{1}{2}$, $\csc\theta = 2$,
$\cos\theta = -\frac{\sqrt{3}}{2}$, and $\sec\theta = -\frac{2}{\sqrt{3}}$

11. For each value of the trigonometric ratio below, determine possible measures of angle θ in the given domain. Give the angles to the nearest degree.

a) $\sin\theta = -\frac{1}{2}$; for $-360° \leq \theta \leq 360°$

Since $\sin\theta$ is negative, the terminal arm of angle θ lies in Quadrant 3 or 4.
For the domain $0° \leq \theta \leq 360°$, from the completed table in question 6,
$\theta = 210°$ or $\theta = 330°$
For the domain $-360° \leq \theta \leq 0°$, $\theta = -150°$ or $\theta = -30°$
The angles are: −150°, −30°, 210°, and 330°

b) $\cot \theta = 1$; for $0° \le \theta \le 720°$

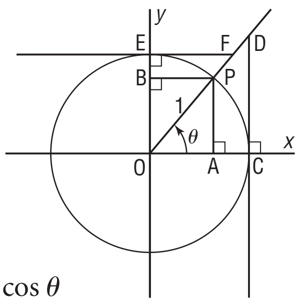
Since $\cot \theta$ is positive, the terminal arm of angle θ lies in Quadrant 1 or 3.
For the domain $0° \le \theta \le 360°$, from the completed table in question 6,
$\theta = 45°$ or $\theta = 225°$
For the domain $360° \le \theta \le 720°$,
$\theta = 45° + 360° = 405°$ or $\theta = 225° + 360° = 585°$
The angles are: 45°, 225°, 405°, and 585°

c) $\sec \theta = -11$; for $0° \le \theta \le 360°$

Since $\sec \theta$ is negative, the terminal arm of angle θ lies in Quadrant 2 or 3.
The reference angle is: $\cos^{-1}\left(\frac{1}{11}\right) \doteq 85°$
For the domain $0° \le \theta \le 360°$:
In Quadrant 2, $\theta \doteq 180° - 85°$, or approximately 95°
In Quadrant 3, $\theta \doteq 180° + 85°$, or approximately 265°
To the nearest degree, the angles are: 95° and 265°

C

12. Perpendicular lines are drawn from the axes to the terminal arm of an angle θ in standard position. Lines DC and EF are tangents to the unit circle. Explain why each trigonometric ratio is equal to the length of the indicated line segment.



a) $PA = \sin \theta$

In right △OPA,
$\sin \theta = \frac{PA}{OP}$
$\sin \theta = \frac{PA}{1}$
So, $PA = \sin \theta$

b) $PB = \cos \theta$

In right △BOP,
$\angle BOP = 90° - \theta$
So, $\angle OPB = \theta$
$\cos \theta = \frac{PB}{OP}$
$\cos \theta = \frac{PB}{1}$
So, $PB = \cos \theta$

c) $OF = \csc \theta$

In right △OEF,
$\angle EFO = \theta$
$\sin \theta = \frac{EO}{OF}$
$\sin \theta = \frac{1}{OF}$
So, $OF = \csc \theta$

d) $DC = \tan \theta$

In right △DOC,
$\tan \theta = \frac{DC}{OC}$
$\tan \theta = \frac{DC}{1}$
So, $DC = \tan \theta$

e) FE $=$ cot θ

In right $\triangle$OEF,

$\angle$EFO $= \theta$

$\tan \theta = \frac{OE}{EF}$

$\tan \theta = \frac{1}{EF}$

So, EF $= \cot \theta$

f) DO $=$ sec θ

In right $\triangle$DOC,

$\cos \theta = \frac{OC}{DO}$

$\cos \theta = \frac{1}{DO}$

So, DO $= \sec \theta$