Lesson 6.3 Exercises, pages 494–501

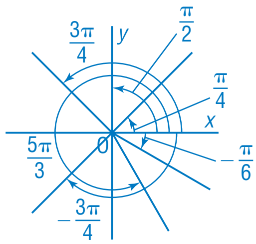
A

4. Sketch each angle in standard position.

a) $\frac{\pi}{4}$ **b)** $\frac{\pi}{2}$

c) $\frac{5\pi}{3}$ **d)** $\frac{3\pi}{4}$

e) $-\frac{\pi}{6}$ **f)** $-\frac{3\pi}{4}$



5. a) Convert each angle to radians.

i) 60°

$= 60\left(\frac{\pi}{180}\right)$

$= \frac{\pi}{3}$

ii) 720°

$= 720\left(\frac{\pi}{180}\right)$

$= 4\pi$

iii) −450°

$= -450\left(\frac{\pi}{180}\right)$

$= -\frac{5\pi}{2}$

b) Convert each angle to degrees. Give the answer to the nearest degree where necessary.

i) 7π

$= 7(180°)$
$= 1260°$

ii) 5

$= 5\left(\frac{180°}{\pi}\right)$
$\doteq 286°$

iii) $-\frac{5\pi}{6}$

$= -\frac{5(180°)}{6}$
$= -150°$

6. Determine the value of each trigonometric ratio to the nearest hundredth.

a) $\cos \frac{\pi}{10}$

$\doteq 0.95$

b) $\sin 2.5$

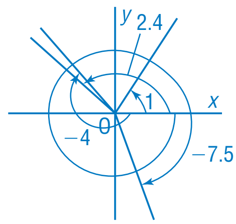
$\doteq 0.60$

c) $\cot\left(-\frac{3\pi}{5}\right)$

$= \frac{1}{\tan\left(-\frac{3\pi}{5}\right)}$
$\doteq 0.32$

B

7. Sketch each angle in radians in standard position.



a) 1

A measure of 1 radian is subtended by an arc of a circle that is approximately $\frac{1}{6}$ of the circumference.

b) 2.4

A measure of 2.4 radians is approximately half way between $\frac{\pi}{2}$ radians and π radians.

c) −4

A measure of −4 radians is subtended by an arc of a circle that is approximately $\frac{4}{6}$ of the circumference.

d) −7.5

A measure of −7.5 radians is subtended by an arc of a circle that is a little longer than $\frac{7}{6}$ of the circumference.

8. Determine the length of the arc that subtends each central angle in a circle with each radius. Give the answers to the nearest tenth where necessary.

a) 3; radius 4 cm

Arc length = (3)(4) cm
= 12 cm

b) 10; radius 5 cm

Arc length = (10)(5) cm
= 50 cm

c) $\frac{\pi}{4}$; radius 6.5 cm

Arc length $= \left(\frac{\pi}{4}\right)(6.5)$ cm
$\doteq 5.1$ cm

d) $\frac{3\pi}{2}$; radius 2.5 cm

Arc length $= \left(\frac{3\pi}{2}\right)(2.5)$ cm
$\doteq 11.8$ cm

9. An arc with length 2 cm is marked on the circumference of a circle with radius 3 cm.

a) To the nearest tenth of a radian, determine the measure of the central angle subtended by the arc.

$$\text{Angle measure} = \frac{\text{arc length}}{\text{radius}}$$
$$= \frac{2}{3}, \text{ or approximately } 0.7$$

b) The area of the sector of a circle is proportional to the central angle. Determine the area of the sector of the circle formed by the arc and the radii that intersect the endpoints of the arc.

$$\frac{\text{Area of sector}}{\text{Area of circle}} = \frac{\frac{2}{3}}{2\pi}$$

$$\text{Area of sector} = \frac{\frac{2}{3}\pi(3)^2}{2\pi} \text{ cm}^2, \text{ or } 3 \text{ cm}^2$$

10. A wheel with diameter 35 cm is rotating at 20 revolutions per minute.

a) What is the speed of the rotating wheel in radians per second? This is the *angular velocity* of the wheel.

20 revolutions per minute is:

$$20(2\pi) \text{ radians per minute} = 40\pi \text{ radians/min}$$
$$= \frac{40}{60}\pi \text{ radians/s, or } \frac{2}{3}\pi \text{ radians/s}$$

The angular velocity is $\frac{2}{3}\pi$ radians/s.

b) Suppose the wheel travels in a straight line. To the nearest centimetre, how far will it travel in 50 s?

$$\text{20 revolutions per minute is: } 20(\pi)(35) \text{ cm/min} = \frac{20(\pi)(35)}{60} \text{ cm/s}$$
$$= \frac{35\pi}{3} \text{ cm/s}$$

So, in 50 s, the wheel will travel: $50\left(\frac{35\pi}{3}\right)$ cm $\doteq 1833$ cm

The wheel travels approximately 18.33 m.

11. For each angle in standard position below:

i) Determine the measures of angles that are coterminal with the angle in the given domain. Give the angles to the nearest tenth where necessary.

ii) Write an expression for the measures of all possible angles that are coterminal with the angle.

a) $\frac{\pi}{3}$; for $-2\pi \leq \theta \leq 2\pi$

i) Coterminal angles are:
$\frac{\pi}{3}$; and $\frac{\pi}{3} - 2\pi = -\frac{5\pi}{3}$

ii) An expression is: $\frac{\pi}{3} + 2\pi k$, $k \in \mathbb{Z}$

b) $\frac{2\pi}{5}$; for $-2\pi \leq \theta \leq 2\pi$

i) Coterminal angles are:
$\frac{2\pi}{5}$; and $\frac{2\pi}{5} - 2\pi = -\frac{8\pi}{5}$

ii) An expression is: $\frac{2\pi}{5} + 2\pi k$, $k \in \mathbb{Z}$

c) 5; for $-2\pi \leq \theta \leq 2\pi$

i) Coterminal angles are:
5 and $5 - 2\pi \doteq -1.3$

ii) An expression is: $5 + 2\pi k$, $k \in \mathbb{Z}$

d) -2; for $-2\pi \leq \theta \leq 2\pi$

i) Coterminal angles are:
-2 and $-2 + 2\pi \doteq 4.3$

ii) An expression is:
$-2 + 2\pi k$, $k \in \mathbb{Z}$

e) $-\frac{7\pi}{6}$; for $-4\pi \leq \theta \leq 4\pi$

i) Coterminal angles are:
$-\frac{7\pi}{6}$; $-\frac{7\pi}{6} - 2\pi = -\frac{19\pi}{6}$;
$-\frac{7\pi}{6} + 2\pi = \frac{5\pi}{6}$; and
$-\frac{7\pi}{6} + 4\pi = \frac{17\pi}{6}$

ii) An expression is: $-\frac{7\pi}{6} + 2\pi k$, $k \in \mathbb{Z}$

f) -8; for $-4\pi \leq \theta \leq 4\pi$

i) Coterminal angles are:
-8; $-8 + 2\pi \doteq -1.7$;
$-8 + 4\pi \doteq 4.6$; and
$-8 + 6\pi \doteq 10.8$

ii) An expression is: $-8 + 2\pi k$, $k \in \mathbb{Z}$

12. Return to Lesson 6.1, question 6, page 476. In the first column, write the equivalent angles in radians.

a) Use the completed table to determine the exact value of each trigonometric ratio.

i) $\sin\frac{\pi}{4}$
$= \frac{1}{\sqrt{2}}$

ii) $\cos\frac{\pi}{6}$
$= \frac{\sqrt{3}}{2}$

iii) $\tan\frac{5\pi}{3}$
$= -\sqrt{3}$

iv) $\sec\frac{\pi}{3}$
$= 2$

b) Determine the exact value of each trigonometric ratio.

i) $\csc 3\pi$

$= \csc \pi$, which is undefined

ii) $\cot\left(-\frac{5\pi}{6}\right)$

$= \cot \frac{\pi}{6}$, or $\sqrt{3}$

iii) $\cos\left(-\frac{7\pi}{3}\right)$

$= \cos\left(-\frac{\pi}{3}\right)$

$= \cos \frac{\pi}{3}$, or $\frac{1}{2}$

iv) $\sin \frac{11\pi}{4}$

$= \sin \frac{3\pi}{4}$

$= \frac{1}{\sqrt{2}}$

c) Describe the strategy you used to complete part b.

For each angle: I identified the quadrant in which its terminal arm lies; I determined the coterminal angle that is in the completed table; then I used the value of its trigonometric ratio.

13. For each point $P(x, y)$ on the terminal arm of an angle θ in standard position

i) Determine the exact values of the 6 trigonometric ratios for θ.

ii) To the nearest tenth of a radian, determine possible values of θ in the domain $-2\pi \leq \theta \leq 2\pi$.

a) $P(1, -2)$

b) $P(-3, 2)$

Let the distance between the origin and P be r. Use: $x^2 + y^2 = r^2$

a)

i) Substitute: $x = 1, y = -2$

$1 + 4 = r^2$

$r = \sqrt{5}$

$\sin \theta = -\frac{2}{\sqrt{5}}$ $\quad \csc \theta = -\frac{\sqrt{5}}{2}$

$\cos \theta = \frac{1}{\sqrt{5}}$ $\quad \sec \theta = \sqrt{5}$

$\tan \theta = -2$ $\quad \cot \theta = -\frac{1}{2}$

ii) The terminal arm of angle θ lies in Quadrant 4.
The reference angle is:
$\tan^{-1}(2) = 1.1071\ldots$
So, $\theta = -1.1071\ldots$
The angle between 0 and 2π that is coterminal with $-1.1071\ldots$ is:
$2\pi - 1.1071\ldots = 5.1760\ldots$
Possible values of θ are approximately: 5.2 and -1.1

b)

i) Substitute: $x = -3, y = 2$

$9 + 4 = r^2$

$r = \sqrt{13}$

$\sin \theta = \frac{2}{\sqrt{13}}$ $\quad \csc \theta = \frac{\sqrt{13}}{2}$

$\cos \theta = -\frac{3}{\sqrt{13}}$ $\quad \sec \theta = -\frac{\sqrt{13}}{3}$

$\tan \theta = -\frac{2}{3}$ $\quad \cot \theta = -\frac{3}{2}$

ii) The terminal arm of angle θ lies in Quadrant 2.
The reference angle is:

$\tan^{-1}\left(\frac{2}{3}\right) = 0.5880\ldots$

So, $\theta = \pi - 0.5880\ldots$
$= 2.5535\ldots$

The angle between -2π and 0 that is coterminal with $2.5535\ldots$ is:
$-2\pi + 2.5535\ldots$
$= -3.7295\ldots$
Possible values of θ are approximately: 2.6 and -3.7

14. For each trigonometric ratio

i) Determine the exact values of the other 5 trigonometric ratios for θ in the given domain.

ii) To the nearest tenth of a radian, determine possible values of θ in the domain $-2\pi \leq \theta \leq 2\pi$.

a) $\cos\theta = \frac{2}{3}$; for $0 \leq \theta \leq \pi$ **b)** $\cot\theta = \frac{3}{5}$; for $\pi \leq \theta \leq 2\pi$

Let P(*x*, *y*) on a circle, radius *r*, be a terminal point of angle θ in standard position.

a)

i) $\cos\theta = \frac{2}{3}$

$\frac{x}{r} = \frac{2}{3}$, so choose $x = 2$ and $r = 3$

Use: $x^2 + y^2 = r^2$

Substitute for x and r.

$4 + y^2 = 9$

$y = \pm\sqrt{5}$

For the given domain, since $\cos\theta$ is positive, the terminal arm of angle θ lies in Quadrant 1 and y is positive.

$\sec\theta = \frac{3}{2}$

$\sin\theta = \frac{\sqrt{5}}{3}$ $\csc\theta = \frac{3}{\sqrt{5}}$

$\tan\theta = \frac{\sqrt{5}}{2}$ $\cot\theta = \frac{2}{\sqrt{5}}$

ii) The reference angle is:

$\cos^{-1}\left(\frac{2}{3}\right) = 0.8410\ldots$

So, $\theta = 0.8410\ldots$

The angle between -2π and 0 that is coterminal with $0.8410\ldots$ is:

$-2\pi + 0.8410\ldots$

$= -5.4421\ldots$

Possible values of θ are approximately: 0.8 and -5.4

b)

i) $\cot\theta = \frac{3}{5}$

For the given domain, since $\cot\theta$ is positive, the terminal arm of angle θ lies in Quadrant 3 where both x and y are negative.

$\frac{x}{y} = \frac{3}{5}$, so choose $x = -3$ and $y = -5$

Use: $x^2 + y^2 = r^2$

Substitute for x and y.

$9 + 25 = r^2$

$r = \sqrt{34}$

$\tan\theta = \frac{5}{3}$

$\sin\theta = -\frac{5}{\sqrt{34}}$ $\csc\theta = -\frac{\sqrt{34}}{5}$

$\cos\theta = -\frac{3}{\sqrt{34}}$ $\sec\theta = -\frac{\sqrt{34}}{3}$

ii) The reference angle is:

$\tan^{-1}\left(\frac{5}{3}\right) = 1.0303\ldots$

So, $\theta = \pi + 1.0303\ldots$

$= 4.1719\ldots$

The angle between -2π and 0 that is coterminal with $4.1719\ldots$ is:

$-2\pi + 4.1719\ldots = -2.1112\ldots$

Possible values of θ are approximately: 4.2 and -2.1

15. Earth completes one orbit of the sun in 1 year. The orbit approximates a circle with radius 150 million kilometres.

a) To the nearest hundred thousand kilometres, what is the length of the arc along Earth's orbit after 30 days?

After 30 days, the length of the arc in kilometres is:

$\frac{30}{365}(2\pi)(150\ 000\ 000) = 77\ 463\ 928.44...$

So, the length of the arc is approximately 77 500 000 km.

b) To the nearest hundred thousand kilometres, what is the straight-line distance between the ends of this arc?

In radians, the central angle subtended by the arc is:

$\frac{30}{365}(2\pi) = 0.5164...$

The straight-line distance is the length of the third side of a triangle with two sides equal to 150 000 000 km and a contained angle of 0.5164...

Use the Cosine Law: $c = \sqrt{t^2 + s^2 - 2ts \cos C}$

Substitute: $t = s = 150\ 000\ 000$, $\angle C = 0.5164...$

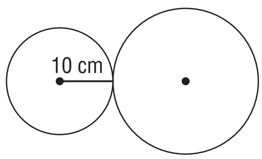
$c = \sqrt{2(150\ 000\ 000)^2 - 2(150\ 000\ 000)^2 \cos 0.5164...}$

$c = 76\ 605\ 988.53...$

The straight-line distance is approximately 76 600 000 km.

C

16. Points on the circumferences of these circles move at the same speed. When the smaller circle rotates through 1.4 radians, the larger circle rotates through 0.9 radians. The radius of the smaller circle is 10 cm. What is the radius of the larger circle, to the nearest tenth of a centimetre?



When the smaller circle rotates through 1.4 radians, a point on the circle traces an arc.

When the larger circle rotates through 0.9 radians, a point on the circle traces an arc.

The lengths of these arcs are equal.

For a rotation of 1.4 radians, the arc length is: (1.4)(10 cm) = 14 cm

Let the radius of the larger circle be r centimetres.

In centimetres, the arc length of the larger circle is: $0.9r$

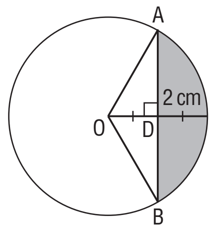
So, $0.9r = 14$

$r = \frac{14}{0.9}$

$r = 15.5555...$

The radius of the larger circle is approximately 15.6 cm.

17. Determine the exact area of the shaded region in this circle, centre O.



The radius of the circle is: 2(2 cm) = 4 cm
In right ΔAOD,
$\cos \angle AOD = \frac{2}{4}$, or $\frac{1}{2}$
So, $\angle AOD = \frac{\pi}{3}$ and $\angle AOB = \frac{2\pi}{3}$
Use the Pythagorean Theorem in ΔAOD.
$2^2 + AD^2 = 4^2$
$AD^2 = 16 - 4$
$AD = \sqrt{12}$
Shaded area = area of sector AOB − area of ΔAOB
$= \frac{\frac{2\pi}{3}}{2\pi}(\pi)(4^2) - \frac{1}{2}(2\sqrt{12})(2)$
$= \frac{16\pi}{3} - 2\sqrt{12}$, or $\frac{16\pi}{3} - 4\sqrt{3}$
The area of the shaded region is $\left(\frac{16\pi}{3} - 4\sqrt{3}\right)$ cm^2.