Checkpoint: Assess Your Understanding, pages 602–604

7.1

1. Multiple Choice How many roots does the equation $\sin 6x = \frac{1}{3}$ have over the domain $0 \leq x < 2\pi$?

A. 2 **B.** 4 **C.** 6 **(D.)** 12

2. Use graphing technology to solve each equation over the given domain. Give the roots to the nearest hundredth.

a) $1 + 2 \sin x = 1 - 3 \cos x$; $0 \leq x \leq 2\pi$

Graph the corresponding function: $y = 2 \sin x + 3 \cos x$
Determine the approximate zeros in the given domain.
The roots are approximately: $x = 2.16$ and $x = 5.30$
Substitute each root into the given equation to verify.

b) $2 = \cos x + 2 \cos^2 x$; $-2\pi \leq x \leq 2\pi$

Graph the corresponding function: $y = \cos x + 2 \cos^2 x - 2$
Determine the approximate zeros in the given domain.
The roots are approximately: $x = \pm 0.67$ and $x = \pm 5.61$
Substitute each root into the given equation to verify.

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3. Use graphing technology to determine the general solution of each equation over the set of real numbers. Give the answers to the nearest hundredth.

a) $4 \tan x - 5 = 0$

Graph the corresponding function: $y = 4 \tan x - 5$
The period of the function is π.
Determine the zero in the domain $0 \leq x < \pi$.
The root is approximately: $x = 0.90$
The general solution is approximately: $x = 0.90 + \pi k, k \in \mathbb{Z}$

b) $6 \cos^2 x + \cos x = 1$

Graph the corresponding function: $y = 6 \cos^2 x + \cos x - 1$
The period of the function is 2π.
Determine the zeros in the domain $0 \leq x < 2\pi$.
The roots are approximately: $x = 1.23$, $x = 2.09$, $x = 4.19$, $x = 5.05$
The general solution is approximately: $x = 1.23 + 2\pi k, k \in \mathbb{Z}$ or $x = 2.09 + 2\pi k, k \in \mathbb{Z}$ or $x = 4.19 + 2\pi k, k \in \mathbb{Z}$ or $x = 5.05 + 2\pi k, k \in \mathbb{Z}$

7.2

4. Multiple Choice Which number is a root of the equation $3 \sin x + 1 = 5 \sin x - 1$ over the domain $0 \leq x < 2\pi$?

A. 0 **B.** π **(C.)** $\frac{\pi}{2}$ **D.** $\frac{3\pi}{2}$

5. Use algebra to solve the equation $\sqrt{2} \cos 2x + 1 = 0$ over the domain $-\pi < x < \pi$, then write the general solution of the equation.

$$\sqrt{2} \cos 2x = -1$$
$$\cos 2x = -\frac{1}{\sqrt{2}}$$

The terminal arm of angle $2x$ lies in Quadrant 2 or 3.

The reference angle for angle $2x$ is: $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

In Quadrant 2, $2x = \frac{3\pi}{4}$, $x = \frac{3\pi}{8}$

In Quadrant 3, $2x = -\frac{3\pi}{4}$, $x = -\frac{3\pi}{8}$

The period of $\cos 2x$ is π, so other roots are:

$x = \frac{3\pi}{8} - \pi$, $x = -\frac{5\pi}{8}$ and $x = -\frac{3\pi}{8} + \pi$, $x = \frac{5\pi}{8}$

The roots are: $x = \pm\frac{3\pi}{8}$ and $x = \pm\frac{5\pi}{8}$

The general solution is: $x = \frac{3\pi}{8} + \pi k, k \in \mathbb{Z}$ or $x = \frac{5\pi}{8} + \pi k, k \in \mathbb{Z}$

6. Verify that $\frac{\pi}{6}$ and $\frac{5\pi}{6}$ are two roots of the equation $4\cos^2 x - 3 = 0$.

Substitute each given value in the equation.

For $x = \frac{\pi}{6}$:

$$\begin{aligned} \text{L.S.} &= 4\cos^2\left(\frac{\pi}{6}\right) - 3 \\ &= 4\left(\frac{\sqrt{3}}{2}\right)^2 - 3 \\ &= 0 \\ &= \text{R.S.} \end{aligned}$$

For $x = \frac{5\pi}{6}$:

$$\begin{aligned} \text{L.S.} &= 4\cos^2\left(\frac{5\pi}{6}\right) - 3 \\ &= 4\left(-\frac{\sqrt{3}}{2}\right)^2 - 3 \\ &= 0 \\ &= \text{R.S.} \end{aligned}$$

For each value of x, the left side is equal to the right side, so the roots are verified.

7. Use algebra to solve the equation $10\sin^2 x + 11\sin x = -3$ over the domain $90° \leq x \leq 360°$. Give the roots to the nearest degree.

$10\sin^2 x + 11\sin x + 3 = 0$
$(2\sin x + 1)(5\sin x + 3) = 0$

Either $2\sin x + 1 = 0$
$\sin x = -0.5$
The reference angle is: $\sin^{-1}(0.5) = 30°$
The terminal arm of angle x lies in Quadrant 3 or 4.
In Quadrant 3, $x = 180° + 30°$, or $210°$
In Quadrant 4, $x = 360° - 30°$, or $330°$

or $5\sin x + 3 = 0$
$\sin x = -0.6$
The reference angle is: $\sin^{-1}(0.6) \doteq 37°$
The terminal arm of angle x lies in Quadrant 3 or 4.
In Quadrant 3, $x \doteq 180° + 37°$, or $217°$
In Quadrant 4, $x \doteq 360° - 37°$, or $323°$

The roots are: $x = 210°$, $x \doteq 217°$, $x \doteq 323°$, $x = 330°$

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