PRACTICE TEST, pages 681–684

1. Multiple Choice What are the roots of the equation $2 \sin x \cos x = 2 \sin^2 x$ over the domain $0 \leq x < 2\pi$?

A. $x = \frac{\pi}{4}, x = \frac{5\pi}{4}$

B. $x = \frac{3\pi}{4}, x = \frac{7\pi}{4}$

C. $x = \frac{\pi}{4}, x = \frac{5\pi}{4}, x = 0, x = \pi$

D. $x = \frac{3\pi}{4}, x = \frac{7\pi}{4}, x = 0, x = \pi$

2. Multiple Choice What is the simplest form of $\cos 5x \sin 2x - \sin 5x \cos 2x$?

A. $-\sin 3x$ **B.** $\sin 3x$ **C.** $-\sin 7x$ **D.** $\sin 7x$

3. Use graphing technology to determine the general solution of the equation $\cot x = \sin x + 1$. Give the answers to the nearest hundredth.

Graph $y = \frac{1}{\tan x} - \sin x - 1$ for $-2\pi \leq x < 2\pi$.
The period of the function is 2π.
The approximate zeros of the function are: −5.708359, −1.570796, 0.5748263, 4.712389
Alternate zeros have a difference of 2π.
The general solution is: $x \doteq 0.57 + 2\pi k, k \in \mathbb{Z}$ or $x \doteq 4.71 + 2\pi k, k \in \mathbb{Z}$

4. Solve the equation $\sin x + 1 = 2 \cos^2 x$ over the domain $-\frac{3\pi}{2} \leq x < \frac{\pi}{2}$. Give the exact roots.

$$\sin x + 1 = 2\cos^2 x$$
$$\sin x + 1 = 2(1 - \sin^2 x)$$
$$2\sin^2 x + \sin x - 1 = 0$$
$$(2\sin x - 1)(\sin x + 1) = 0$$

Either $2\sin x - 1 = 0$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}$ **or** $x = -\frac{7\pi}{6}$

or $\sin x + 1 = 0$

$\sin x = -1$

$x = -\frac{\pi}{2}$

The roots are: $x = -\frac{7\pi}{6}, x = -\frac{\pi}{2}$, **and** $x = \frac{\pi}{6}$

5. Determine the general solution of the equation $\sin 4x = \frac{1}{\sqrt{2}}$ over the set of real numbers.

$\sin 4x = \frac{1}{\sqrt{2}}$

$4x = \frac{\pi}{4}$ or $4x = \frac{3\pi}{4}$

$x = \frac{\pi}{16}$ $\quad x = \frac{3\pi}{16}$

The period of $\sin 4x$ is $\frac{2\pi}{4}$ or $\frac{\pi}{2}$.

So, the general solution is: $x = \frac{\pi}{16} + \frac{\pi}{2}k, k \in \mathbb{Z}$ or $x = \frac{3\pi}{16} + \frac{\pi}{2}k, k \in \mathbb{Z}$

6. For the identity $\frac{\cos\theta + \cot\theta}{1 + \sin\theta} = \cot\theta$

a) Determine the non-permissible values of θ.

Non-permissible values occur when: $\sin\theta = 0$, $\theta = \pi k, k \in \mathbb{Z}$ or

$1 + \sin\theta = 0$

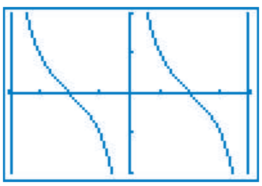
$\sin\theta = -1$

$\theta = \frac{\pi}{2} + \pi k, k \in \mathbb{Z}$

b) Verify the identity graphically. Sketch or print the graph and explain how it verifies the identity.

Graph $y = \frac{\cos\theta + \frac{\cos\theta}{\sin\theta}}{1 + \sin\theta}$ and $y = \frac{\cos\theta}{\sin\theta}$.

The graphs coincide so the identity is verified.



c) Verify the identity for $\theta = \frac{\pi}{3}$. Explain why this verification does not prove the identity.

Substitute $\theta = \frac{\pi}{3}$ in each side of the identity.

$$\text{L.S.} = \frac{\cos\theta + \cot\theta}{1 + \sin\theta} = \frac{\cos\frac{\pi}{3} + \cot\frac{\pi}{3}}{1 + \sin\frac{\pi}{3}} = \frac{\left(\frac{1}{2}\right) + \left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{\sqrt{3}}{2}\right)} = \frac{\frac{\sqrt{3} + 2}{2\sqrt{3}}}{\frac{2 + \sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

$$\text{R.S.} = \cot\theta = \cot\frac{\pi}{3} = \frac{1}{\sqrt{3}}$$

The left side is equal to the right side, so the identity is verified. This verification does not prove the identity because there may be values of θ, apart from the non-permissible values, for which the left side does not equal the right side.

d) Prove the identity.

$$\text{L.S.} = \frac{\cos\theta + \cot\theta}{1 + \sin\theta} = \frac{\cos\theta + \frac{\cos\theta}{\sin\theta}}{1 + \sin\theta} = \frac{\sin\theta\cos\theta + \cos\theta}{(\sin\theta)(1 + \sin\theta)} = \frac{(\cos\theta)(\sin\theta + 1)}{(\sin\theta)(1 + \sin\theta)} = \frac{\cos\theta}{\sin\theta} = \cot\theta = \text{R.S.}$$

The left side is equal to the right side, so the identity is proved.

7. Given angle α in standard position with its terminal arm in Quadrant 3 and $\sin \alpha = -\frac{2}{3}$, and angle β in standard position with its terminal arm in Quadrant 4 and $\cos \beta = \frac{3}{7}$, determine each exact value.

a) $\cos(\alpha - \beta)$

Use: $x^2 + y^2 = r^2$

For angle α, substitute:

$y = -2, r = 3$

$x^2 + (-2)^2 = 3^2$

$x = \pm\sqrt{5}$

$x = -\sqrt{5}$ since the terminal arm of angle α lies in Quadrant 3.

So, $\cos \alpha = -\frac{\sqrt{5}}{3}$

Substitute for α and β in:

$\cos(\alpha - \beta)$

$= \cos \alpha \cos \beta + \sin \alpha \sin \beta$

$= \left(-\frac{\sqrt{5}}{3}\right)\left(\frac{3}{7}\right) + \left(-\frac{2}{3}\right)\left(-\frac{\sqrt{40}}{7}\right)$

$= \frac{-3\sqrt{5} + 2\sqrt{40}}{21}$

b) $\tan 2\alpha$

For angle β, substitute:

$x = 3, r = 7$

$3^2 + y^2 = 7^2$

$y = \pm\sqrt{40}$

$y = -\sqrt{40}$ since the terminal arm of angle β lies in Quadrant 4.

So, $\sin \beta = -\frac{\sqrt{40}}{7}$

$\tan 2\alpha = \frac{\sin 2\alpha}{\cos 2\alpha}$

$= \frac{2 \sin \alpha \cos \alpha}{\cos^2 \alpha - \sin^2 \alpha}$

$= \frac{2\left(-\frac{2}{3}\right)\left(-\frac{\sqrt{5}}{3}\right)}{\left(-\frac{\sqrt{5}}{3}\right)^2 - \left(-\frac{2}{3}\right)^2}$, or $4\sqrt{5}$

8. Prove this identity: $\frac{2 - 2\cos 2\theta}{2 \sin 2\theta} = \frac{\sec^2\theta - 1}{\tan \theta}$

L.S. $= \frac{2 - 2\cos 2\theta}{2 \sin 2\theta}$

$= \frac{1 - \cos 2\theta}{\sin 2\theta}$

$= \frac{1 - (1 - 2\sin^2\theta)}{2 \sin \theta \cos \theta}$

$= \frac{2 \sin^2\theta}{2 \sin \theta \cos \theta}$

$= \frac{\sin \theta}{\cos \theta}$

R.S. $= \frac{\sec^2\theta - 1}{\tan \theta}$

$= \frac{\tan^2\theta}{\tan \theta}$

$= \tan \theta$

$= \frac{\sin \theta}{\cos \theta}$

The left side and the right side simplify to the same expression, so the identity is proved.