

Lesson 8.1 Exercises, pages 689–693

A

3. A gift-wrapping booth has 3 sizes of boxes, 2 colours of gift wrap, and 4 different bows. How many choices of wrapping are possible using a box, gift wrap, and bow?

Use the fundamental counting principle.

There are 3 ways to choose a box.

There are 2 ways to choose the gift wrap.

There are 4 ways to choose a bow.

So, the number of choices of wrapping is: $3 \cdot 2 \cdot 4 = 24$

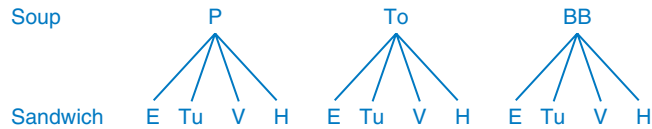
So, 24 choices of wrapping are possible.

4. A school cafeteria offers a soup and sandwich combo. There are 3 kinds of soup (pea, tomato, black bean) and 4 kinds of sandwiches (egg salad, tuna, veggie, ham). Use a graphic organizer to show the number of possible combos.

Use a tree diagram. Use letters to represent the cafeteria items.

Soups: P = pea, To = tomato, BB = black bean

Sandwiches: E = egg salad, Tu = tuna, V = veggie, H = ham



The tree diagram has 12 branches. So, there are 12 possible combos.

5. a) How many Alberta license plates were possible in 1912? Assume there were no restrictions on the digits.



For each digit, there are 10 choices.

So, the number of possible plates is: $10 \cdot 10 \cdot 10 \cdot 10 = 10\,000$

So, 10 000 license plates were possible.

- b) Suppose 0 is not permitted as the first digit. How many license plates were possible?

If 0 is not permitted as the first digit, there are only 9 choices for the first digit. For every other digit, there are 10 choices.

So, the number of possible plates is: $9 \cdot 10 \cdot 10 \cdot 10 = 9000$

So, 9000 license plates were possible.

B

6. Use the digits 5, 6, 7, and 8.

- a) How many 4-digit numbers can be formed when repetition is allowed?

Use the fundamental counting principle.

There are 4 digits to choose from.

There are 4 ways to choose each digit.

So, the number of 4-digit numbers is: $4 \cdot 4 \cdot 4 \cdot 4 = 256$

So, 256 four-digit numbers can be formed.

- b) How many 4-digit numbers can be formed when repetition is not allowed?

Use the fundamental counting principle.

There are 4 ways to choose the 1st digit, 3 ways to choose the 2nd digit, 2 ways to choose the 3rd digit, and 1 way to choose the last digit.

So, the number of 4-digit numbers is: $4 \cdot 3 \cdot 2 \cdot 1 = 24$

So, 24 four-digit numbers can be formed.

7. How many 2-digit numbers less than 60 are even? Verify your work using another strategy.

Use an organized list.

Tens digit	Possible even numbers
1	10, 12, 14, 16, 18
2	20, 22, 24, 26, 28
3	30, 32, 34, 36, 38
4	40, 42, 44, 46, 48
5	50, 52, 54, 56, 58

So, 25 two-digit numbers less than 60 are even.

Verify.

There are 5 ways to choose the 1st digit: 1, 2, 3, 4, 5

For each of these digits, there are 5 ways to choose the 2nd digit:

0, 2, 4, 6, 8

So, the number of 2-digit even numbers less than 60 is: $5 \cdot 5 = 25$

8. In the women's gold-medal hockey game at the 2010 Vancouver Olympics, Team Canada defeated Team U.S.A. 2 – 0. Each team had 21 players on its roster. Assume each of the 21 players on Team Canada shook hands with each of the 21 players on Team U.S.A. What was the total number of handshakes?

Use the fundamental counting principle.

There are 21 ways to choose a player on Team Canada.

There are 21 ways to choose a player on Team U.S.A.

So, the number of possible choices is:

$$21 \cdot 21 = 441$$

So, the total number of handshakes was 441.

9. A mobile phone has an 8-digit code called a PIN Unlock Key (PUK). This code is used to unlock a phone when an incorrect PIN number has been used three times in succession. All digits can be used. How many PUK codes are possible?

Use the fundamental counting principle.

For each digit, there are 10 choices.

The number of PUK codes possible is:

$$10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 100\,000\,000$$

So, 100 000 000 PUK codes are possible.

- 10.** How many radio call letters beginning with C and consisting of 4 letters can be assigned to radio stations in Canada when repetition is not allowed?

Use the fundamental counting principle.

There is only 1 choice for the 1st letter: C

There are $26 - 1$, or 25 choices for the 2nd letter.

There are $25 - 1$, or 24 choices for the 3rd letter.

There are $24 - 1$, or 23 choices for the 4th letter.

So, the number of radio call letters is:

$$1 \cdot 25 \cdot 24 \cdot 23 = 13\,800$$

- 11.** A pizza chain offers cheese pizzas with a choice of these toppings: pepperoni, onion, sausage, mushrooms, and anchovies.

- a) Explain why the pizza chain can claim to offer 32 different pizzas.

There are 5 toppings offered.

A pizza can have or not have each topping.

So, the number of different pizzas is: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$

- b) Another pizza chain claims that with its choice of toppings, it can create just over 1000 pizzas. What is the minimum number of toppings it must offer?

Let x represent the number of toppings offered.

A pizza can have or not have each topping.

The number of different pizzas is: 2^x

Since the pizza chain can create just over 1000 pizzas, let $2^x = 1000$.

Since $2^{10} = 1024$, which is close to 1000, $x = 10$

The pizza chain must offer a minimum of 10 toppings.

- 12.** There are 700 students in a high school. Explain why at least two students must have the same first initial and the same last initial.

For each student, there are 26 choices for the first initial and 26 choices for the last initial.

So, the number of possible choices of first and last initials is: $26 \cdot 26 = 676$

Since there are 700 students in the school and $676 < 700$, at least 2 students must have the same first initial and the same last initial.

- 13.** Which event is more likely? Why?

- Tossing 23 tails with 23 pennies
- Rolling 9 sixes with 9 dice

The probability of tossing a tail with a penny is $\frac{1}{2}$.

So, the probability of tossing 23 tails with 23 pennies is: $\left(\frac{1}{2}\right)^{23} = \frac{1}{8\,388\,608}$

The probability of rolling a six with a die is $\frac{1}{6}$.

So, the probability of rolling 9 sixes with 9 dice is: $\left(\frac{1}{6}\right)^9 = \frac{1}{10\,077\,696}$

Since $\frac{1}{10\,077\,696} < \frac{1}{8\,388\,608}$, tossing 23 tails with 23 pennies is more likely.

C

- 14.** A die has faces labelled 1 to 6. The number of outcomes when n dice are rolled is 279 936. How many dice were rolled? Explain your reasoning.

For each die, there are 6 possible outcomes.

For n dice, there are $6 \cdot 6 \cdot 6 \cdot \dots \cdot 6$ (n times), or 6^n possible outcomes.

Let $6^n = 279\,936$

$$6^n = 6^7$$

$$n = 7$$

So, 7 dice were rolled.

- 15.** Determine the number of ways of rolling 4 or 7 with two dice labelled from 1 to 6. Explain why you cannot use the fundamental counting principle.

The number of ways to roll a 4 with 2 dice:

$$1 + 3 \quad 3 + 1 \quad 2 + 2$$

So, there are 3 ways to roll a 4.

The number of ways to roll a 7 with 2 dice:

$$1 + 6 \quad 6 + 1 \quad 5 + 2 \quad 2 + 5 \quad 3 + 4 \quad 4 + 3$$

So, there are 6 ways to roll a 7.

So, the number of ways of rolling 4 or 7 with 2 dice is $3 + 6 = 9$.

I can't use the fundamental counting principle because the ways I get a sum of 4 are not related to or dependent on the ways I get a sum of 7. I add the numbers of ways instead of multiplying.

- 16.** Three couples go to see a movie at Cinematheque in Winnipeg. They sit together in 6 consecutive seats and couples sit together. How many seating arrangements are possible?

There are 6 consecutive seats.

Any one of the 6 people can sit in the first seat.

Her or his partner must sit in the second seat.

That leaves any one of 4 people to sit in the third seat.

Her or his partner must sit in the fourth seat.

That leaves one of 2 people to sit in the fifth seat.

Her or his partner must sit in the sixth seat.

So, the number of seating arrangements is: $6 \cdot 1 \cdot 4 \cdot 1 \cdot 2 \cdot 1 = 48$