Lesson 1.1 Exercises, pages 8–12

A

4. Circle each sequence that could be arithmetic. Determine its common difference, d.

a) 6, 10, 14, 18, . . .

d is: $10 - 6 = 4$

b) 9, 7, 5, 3, . . .

d is: $7 - 9 = -2$

c) −11, −4, 3, 10, . . .

d is: $-4 - (-11) = 7$

d) 2, −4, 8, −16, . . .

Not arithmetic

5. Each sequence is arithmetic. Determine each common difference, d, then list the next 3 terms.

a) 12, 15, 18, . . .

d is: $15 - 12 = 3$
The next 3 terms are:
$18 + 3, 18 + 6, 18 + 9$;
or 21, 24, 27

b) 25, 21, 17, . . .

d is: $21 - 25 = -4$
The next 3 terms are:
$17 - 4, 17 - 8, 17 - 12$;
or 13, 9, 5

6. Determine the indicated term of each arithmetic sequence.

a) 6, 11, 16, . . . ; t_7

Use: $t_n = t_1 + d(n - 1)$
Substitute: $n = 7, t_1 = 6, d = 5$
$t_7 = 6 + 5(7 - 1)$
$t_7 = 36$

b) $2, 1\frac{1}{2}, 1, \ldots; t_{35}$

Use: $t_n = t_1 + d(n - 1)$
Substitute: $n = 35, t_1 = 2, d = -\frac{1}{2}$
$t_{35} = 2 - \frac{1}{2}(35 - 1)$
$t_{35} = -15$

7. Write the first 4 terms of each arithmetic sequence, given the first term and the common difference.

a) $t_1 = -3, d = 4$

$t_1 = -3$
t_2 is $t_1 + d = -3 + 4$, or 1
t_3 is $t_2 + d = 1 + 4$, or 5
t_4 is $t_3 + d = 5 + 4$, or 9

b) $t_1 = -0.5, d = -1.5$

$t_1 = -0.5$
t_2 is $t_1 + d = -0.5 - 1.5$, or -2
t_3 is $t_2 + d = -2 - 1.5$, or -3.5
t_4 is $t_3 + d = -3.5 - 1.5$, or -5

B

8. When you know the first term and the common difference of an arithmetic sequence, how can you tell if it is increasing or decreasing? Use examples to explain.

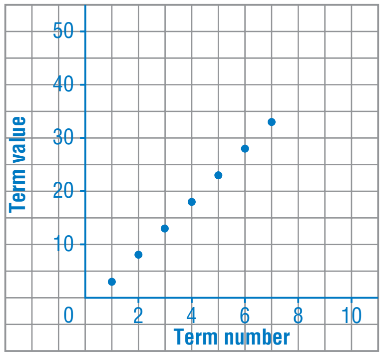
An arithmetic sequence is increasing if d is positive; for example, when t_1 is -10 and $d = 3$: −10, −7, −4, −1, 2, . . .
An arithmetic sequence is decreasing if d is negative; for example, when t_1 is -10 and $d = -3$: −10, −13, −16, −19, −22, . . .

9. a) Create your own arithmetic sequence. Write the first 7 terms. Explain your method.

Sample response: I chose $t_1 = 3$ and $d = 5$; I add 5 to 3, then keep adding 5. The first 7 terms of the sequence are: 3, 8, 13, 18, 23, 28, 33

b) Use technology or grid paper to graph the sequence in part a. Plot the *Term value* on the vertical axis and the *Term number* on the horizontal axis. Print the graph or sketch it on this grid.

Sample response



i) How do you know that the graph represents a linear function?

The points lie on a non-vertical straight line.

ii) What does the slope of the line through the points represent? Explain why.

The slope is the common difference because it is the rise when the run is 1; that is, after the first point, each point can be plotted by moving 5 units up and 1 unit right.

10. Two terms of an arithmetic sequence are given. Determine the indicated terms.

a) $t_4 = 24, t_{10} = 66$; determine t_1

$t_{10} = t_4 + 6d$
Substitute for t_{10} and t_4, then solve for d.
$66 = 24 + 6d$
$6d = 42$
$d = 7$
$t_1 = t_4 - 3d$
Substitute for t_4 and d.
$t_1 = 24 - 3(7)$
$t_1 = 3$

b) $t_3 = 81, t_{12} = 27$; determine t_{23}

$t_{12} = t_3 + 9d$
Substitute for t_{12} and t_3.
$27 = 81 + 9d$
$9d = -54$
$d = -6$
$t_{23} = t_{12} + 11d$
Substitute for t_{12} and d.
$t_{23} = 27 + 11(-6)$
$t_{23} = -39$

11. Create an arithmetic sequence for each description below. For each sequence, write the first 6 terms and a rule for t_n.

a) an increasing sequence

Sample response:
Choose a positive common difference.
Use: $t_1 = 4$ and $d = 3$
The sequence is:
4, 7, 10, 13, 16, 19, . . .
Use: $t_n = t_1 + d(n - 1)$
Substitute: $t_1 = 4, d = 3$
$t_n = 4 + 3(n - 1)$
$t_n = 1 + 3n$

b) a decreasing sequence

Choose a negative common difference.
Use: $t_1 = 4$ and $d = -3$
The sequence is:
4, 1, −2, −5, −8, −11, . . .
Use: $t_n = t_1 + d(n - 1)$
Substitute: $t_1 = 4, d = -3$
$t_n = 4 - 3(n - 1)$
$t_n = 7 - 3n$

c) every term is negative

Sample response:

Choose a negative first term, and a negative common difference.

Use: $t_1 = -2$ and $d = -3$

The sequence is:

$-2, -5, -8, -11, -14, -17, \ldots$

Use: $t_n = t_1 + d(n - 1)$

Substitute: $t_1 = -2, d = -3$

$t_n = -2 - 3(n - 1)$

$t_n = 1 - 3n$

d) every term is an even number

Choose an even first term and an even common difference.

Use: $t_1 = -2$ and $d = -4$

The sequence is:

$-2, -6, -10, -14, -18, -22, \ldots$

Use: $t_n = t_1 + d(n - 1)$

Substitute: $t_1 = -2, d = -4$

$t_n = -2 - 4(n - 1)$

$t_n = 2 - 4n$

12. Claire wrote the first 3 terms of an arithmetic sequence: 3, 6, 9, . . .
When she asked Alex to extend the sequence to the first 10 terms, he wrote:

3, 6, 9, 3, 6, 9, 3, 6, 9, 3, . . .

a) Is Alex correct? Explain.

No, Alex's sequence is not arithmetic because the terms do not increase or decrease by the same number.

b) What fact did Alex ignore when he extended the sequence?

Alex did not use the common difference of 3 to calculate each term.

c) What is the correct sequence?

Add 3 to get each next term: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, . . .

13. Determine whether 100 is a term of an arithmetic sequence with $t_3 = 250$ and $t_6 = 245.5$.

Let the common difference be d.

Use: $t_6 = t_3 + 3d$ Substitute: $t_6 = 245.5, t_3 = 250$

$245.5 = 250 + 3d$

$3d = -4.5$

$d = -1.5$

Use: $t_1 = t_3 - 2d$ Substitute: $t_3 = 250, d = -1.5$

$t_1 = 250 - 2(-1.5)$

$t_1 = 253$

Use: $t_n = t_1 + d(n - 1)$ Substitute: $t_n = 100, d = -1.5, t_1 = 253$

$100 = 253 - 1.5(n - 1)$

$1.5n = 154.5$

$n = 103$

Since $t_{103} = 100$, then 100 is a term of the sequence.

14. The Chinese zodiac associates years with animals. Ling was born in 1994, the Year of the Dog.

a) The Year of the Dog repeats every 12 years. List the first three years that Ling will celebrate her birthday in the Year of the Dog.

Add 12 to 1994 three times: 2006, 2018, 2030

b) Why do the years in part a form an arithmetic sequence?

The difference between consecutive dates is constant.

c) In 2099, Nunavut will celebrate its 100th birthday.

Will that year also be the Year of the Dog? Explain.

All terms in the sequence are even numbers;
2099 is an odd number so 2099 cannot be the Year of the Dog.

15. In this arithmetic sequence: 3, 8, 13, 18, . . . ; which term has the value 123?

d is: $8 - 3 = 5$

Use: $t_n = t_1 + d(n - 1)$ Substitute: $t_n = 123$, $t_1 = 3$, $d = 5$

$123 = 3 + 5(n - 1)$

$123 = 3 + 5n - 5$

$5n = 125$

$n = 25$

123 is the 25th term.

16. For two different arithmetic sequences, $t_5 = -1$. What are two possible sequences? Explain your reasoning.

Sample response: For one sequence, choose a number for the common difference, *d*, such as 5. Keep subtracting 5 to get the preceding terms.

$t_5 = -1$ $\quad t_4 = -1 - 5 = -6$ $\quad t_3 = -6 - 5 = -11$ $\quad t_2 = -11 - 5 = -16$ $\quad t_1 = -16 - 5 = -21$

One arithmetic sequence is: −21, −16, −11, −6, −1, . . .

For the other sequence, choose a different number for *d*, such as −8. Keep subtracting −8.

$t_5 = -1$ $\quad t_4 = -1 + 8 = 7$ $\quad t_3 = 7 + 8 = 15$ $\quad t_2 = 15 + 8 = 23$ $\quad t_1 = 23 + 8 = 31$

Another arithmetic sequence is: 31, 23, 15, 7, −1, . . .

C

17. A sequence is created by adding each term of an arithmetic sequence to the preceding term.

a) Show that the new sequence is arithmetic.

Use the general sequence: $t_1, t_1 + d, t_1 + 2d, t_1 + 3d, t_1 + 4d, \dots$
The new sequence is: $t_1 + t_1 + d, t_1 + d + t_1 + 2d, t_1 + 2d + t_1 + 3d,$
$t_1 + 3d + t_1 + 4d, \dots$
This simplifies to: $2t_1 + d, 2t_1 + 3d, 2t_1 + 5d, 2t_1 + 7d, \dots$
This sequence has first term $2t_1 + d$ and common difference $2d$, so the sequence is arithmetic.

b) How are the common differences of the two sequences related?

The common difference of the new sequence is double the common difference of the original sequence.

18. In this arithmetic sequence, k is a natural number: $k, \frac{2k}{3}, \frac{k}{3}, 0, \dots$

a) Determine t_6.

The common difference, d, is: $\frac{2k}{3} - k = -\frac{k}{3}$

$t_4 = 0 \quad t_5 = 0 + \left(-\frac{k}{3}\right) \quad t_6 = -\frac{k}{3} - \frac{k}{3}$

$\qquad\qquad\quad = -\frac{k}{3} \qquad\qquad\quad = -\frac{2k}{3}$

t_6 is $-\frac{2k}{3}$.

b) Write an expression for t_n.

Use: $t_n = t_1 + d(n - 1)$ Substitute: $t_1 = k, d = -\frac{k}{3}$

$t_n = k + \left(-\frac{k}{3}\right)(n - 1)$

$t_n = k - \frac{kn}{3} + \frac{k}{3}$

$t_n = \frac{4k}{3} - \frac{kn}{3}$

c) Suppose $t_{20} = -16$; determine the value of k.

Use: $t_n = \frac{4k}{3} - \frac{kn}{3}$ Substitute: $t_n = -16, n = 20$

$-16 = \frac{4k}{3} - \frac{20k}{3}$

$-16 = -\frac{16k}{3}$

$k = 3$