Lesson 1.2 Exercises, pages 19–24

A

3. Use each arithmetic sequence to write the first 4 terms of an arithmetic series.

a) 2, 4, 6, 8, . . .

$2 + 4 + 6 + 8$

b) $-2, 3, 8, 13, \ldots$

$-2 + 3 + 8 + 13$

c) $4, 0, -4, -8, \ldots$

$4 + 0 - 4 - 8$

d) $\frac{1}{2}, \frac{1}{4}, 0, -\frac{1}{4}, \ldots$

$\frac{1}{2} + \frac{1}{4} + 0 - \frac{1}{4}$

4. Determine the sum of the given terms of each arithmetic series.

a) $12 + 10 + 8 + 6 + 4$

$12 + 10 + 8 + 6 + 4 = 40$

b) $-2 - 4 - 6 - 8 - 10$

$-2 - 4 - 6 - 8 - 10 = -30$

5. Determine the sum of the first 20 terms of each arithmetic series.

a) $3 + 7 + 11 + 15 + \ldots$

Use: $S_n = \frac{n[2t_1 + d(n - 1)]}{2}$

Substitute:

$n = 20, t_1 = 3, d = 4$

$S_{20} = \frac{20[2(3) + 4(20 - 1)]}{2}$

$S_{20} = 820$

b) $-21 - 15.5 - 10 - 4.5 - \ldots$

Use: $S_n = \frac{n[2t_1 + d(n - 1)]}{2}$

Substitute:

$n = 20, t_1 = -21, d = 5.5$

$S_{20} = \frac{20[2(-21) + 5.5(20 - 1)]}{2}$

$S_{20} = 625$

B

6. For each arithmetic series, determine the indicated value.

a) $-4 - 11 - 18 - 25 - \ldots$; determine S_{28}

Use: $S_n = \frac{n[2t_1 + d(n - 1)]}{2}$

Substitute:

$n = 28, t_1 = -4, d = -7$

$S_{28} = \frac{28[2(-4) - 7(28 - 1)]}{2}$

$S_{28} = -2758$

b) $1 + 3.5 + 6 + 8.5 + \ldots$; determine S_{42}

Use: $S_n = \frac{n[2t_1 + d(n - 1)]}{2}$

Substitute:

$n = 42, t_1 = 1, d = 2.5$

$S_{42} = \frac{42[2(1) + 2.5(42 - 1)]}{2}$

$S_{42} = 2194.5$

7. Use the given data about each arithmetic series to determine the indicated value.

a) $S_{20} = -850$ and $t_{20} = -90$; determine t_1

Use: $S_n = \dfrac{n(t_1 + t_n)}{2}$

Substitute:

$S_n = -850, n = 20, t_n = -90$

$$-850 = \frac{20(t_1 - 90)}{2}$$

$$-850 = 10t_1 - 900$$

$$50 = 10t_1$$

$$t_1 = 5$$

b) $S_{15} = 322.5$ and $t_1 = 4$; determine d

Use: $S_n = \dfrac{n[2t_1 + d(n - 1)]}{2}$

Substitute:

$S_n = 322.5, n = 15, t_1 = 4$

$$322.5 = \frac{15[2(4) + d(15 - 1)]}{2}$$

$$322.5 = 7.5(8 + 14d)$$

$$43 = 8 + 14d$$

$$35 = 14d$$

$$d = 2.5$$

c) $S_n = -126$, $t_1 = -1$, and $t_n = -20$; determine n

Use: $S_n = \dfrac{n(t_1 + t_n)}{2}$

Substitute:

$S_n = -126, t_1 = -1, t_n = -20$

$$-126 = \frac{n(-1 - 20)}{2}$$

$$-252 = -21n$$

$$n = 12$$

d) $t_1 = 1.5$ and $t_{20} = 58.5$; determine S_{15}

Use $t_n = t_1 + d(n - 1)$ **to determine** d**.**

Substitute: $t_n = 58.5, t_1 = 1.5, n = 20$

$$58.5 = 1.5 + d(20 - 1)$$

$$57 = 19d$$

$$d = 3$$

Use: $S_n = \dfrac{n[2t_1 + d(n - 1)]}{2}$

Substitute: $n = 15, t_1 = 1.5, d = 3$

$$S_{15} = \frac{15[2(1.5) + 3(15 - 1)]}{2}$$

$$S_{15} = \frac{15(3 + 42)}{2}$$

$$S_{15} = 337.5$$

8. Two hundred seventy-six students went to a powwow. The first bus had 24 students. The numbers of students on the buses formed an arithmetic sequence. What additional information do you need to determine the number of buses? Explain your reasoning.

I need to know the number of students on the last bus, then I can use the rule $S_n = \dfrac{n(t_1 + t_n)}{2}$**. Or, I need to know the common difference,** d**, then I can use the rule** $S_n = \dfrac{n[2t_1 + d(n - 1)]}{2}$**. In each rule, I substitute what I know, then solve for** n**.**

9. Ryan's grandparents loaned him the money to purchase a BMX bike. He agreed to repay \$25 at the end of the first month, \$30 at the end of the second month, \$35 at the end of the third month, and so on. Ryan repaid the loan in 12 months. How much did the bike cost? How do you know your answer is correct?

Ryan's repayments form an arithmetic series with 12 terms, where the 1st term is his first payment, and the common difference is \$5.

Use: $S_n = \frac{n[2t_1 + d(n - 1)]}{2}$ **Substitute:** $n = 12$, $t_1 = 25$, $d = 5$

$S_{12} = \frac{12[2(25) + 5(12 - 1)]}{2}$

$S_{12} = 6(50 + 55)$

$S_{12} = 630$

The bike cost \$630.

I used a calculator to add the 12 payments to check that the answer is the same.

10. Determine the sum of the indicated terms of each arithmetic series.

a) $31 + 35 + 39 + \ldots + 107$

Use $t_n = t_1 + d(n - 1)$ **to determine** n**. Substitute:**
$t_n = 107$, $t_1 = 31$, **and** $d = 4$

$107 = 31 + 4(n - 1)$

$76 = 4n - 4$

$80 = 4n$

$n = 20$

Use: $S_n = \frac{n(t_1 + t_n)}{2}$

Substitute:
$n = 20$, $t_1 = 31$, **and** $t_n = 107$

$S_{20} = \frac{20(31 + 107)}{2}$

$S_{20} = 1380$

b) $-13 - 10 - 7 - \ldots + 62$

Use $t_n = t_1 + d(n - 1)$ **to determine** n**. Substitute:**
$t_n = 62$, $t_1 = -13$, **and** $d = 3$

$62 = -13 + 3(n - 1)$

$75 = 3n - 3$

$78 = 3n$

$n = 26$

Use: $S_n = \frac{n(t_1 + t_n)}{2}$

Substitute:
$n = 26$, $t_1 = -13$, **and** $t_n = 62$

$S_{26} = \frac{26(-13 + 62)}{2}$

$S_{26} = 637$

11. a) Explain how this series could be arithmetic.
$1 + 3 + \ldots$

This series could be arithmetic if each term was calculated by adding 2 to the preceding term.

b) What information do you need to be certain that this is an arithmetic series?

I need to know that the number added each time is 2.

12. An arithmetic series has $S_{10} = 100$, $t_1 = 1$, and $d = 2$. How can you use this information to determine S_{11} without using a rule for the sum of an arithmetic series? What is S_{11}?

$S_{11} = S_{10} + t_{11}$
I use $t_n = t_1 + d(n - 1)$ to determine t_{11}.
I substitute $n = 11$, $t_1 = 1$, $d = 2$.
$t_{11} = 1 + 2(11 - 1)$
$t_{11} = 21$
Then $S_{11} = 100 + 21$
$S_{11} = 121$

13. The side lengths of a quadrilateral form an arithmetic sequence. The perimeter is 74 cm. The longest side is 29 cm. What are the other side lengths?

The sum of the side lengths form an arithmetic series with 4 terms, where $t_4 = 29$ and $S_4 = 74$.

Use $S_n = \frac{n(t_1 + t_n)}{2}$ to determine t_1.

Substitute: $S_n = 74$, $n = 4$, $t_n = 29$

$74 = \frac{4(t_1 + 29)}{2}$

$74 = 2t_1 + 58$
$16 = 2t_1$
$t_1 = 8$

Use $t_n = t_1 + d(n - 1)$ to determine d.
Substitute: $t_n = 29$, $t_1 = 8$, $n = 4$
$29 = 8 + d(4 - 1)$
$21 = 3d$
$d = 7$
So, the other side lengths are: 8 cm, 15 cm, and 22 cm

14. Derive a rule for the sum of the first n natural numbers:
$1 + 2 + 3 + \ldots + n$

The sum of the numbers is an arithmetic series with $t_1 = 1$, $d = 1$, and $t_n = n$.

Use: $S_n = \frac{n[2t_1 + d(n - 1)]}{2}$ Substitute: $t_1 = 1$, $d = 1$

$S_n = \frac{n[2(1) + 1(n - 1)]}{2}$

$S_n = \frac{n(n + 1)}{2}$

15. The sum of the first 5 terms of an arithmetic series is 170. The sum of the first 6 terms is 225. The common difference is 7. Determine the first 4 terms of the series.

$S_5 = 170, S_6 = 225$
$t_6 = S_6 - S_5$
$= 225 - 170$
$= 55$
Use $t_n = t_1 + d(n - 1)$ to determine t_1.
Substitute: $t_n = 55, d = 7, n = 6$
$55 = t_1 + 7(6 - 1)$
$t_1 = 20$
So, $t_2 = 27, t_3 = 34$, and $t_4 = 41$
The first 4 terms are: $20 + 27 + 34 + 41$

C

16. The sum of the first n terms of an arithmetic series is: $S_n = 3n^2 - 8n$
Determine the first 4 terms of the series.

Determine S_1, S_2, S_3, and S_4.
In $S_n = 3n^2 - 8n$:

Substitute: $n = 1$
$S_1 = 3(1)^2 - 8(1)$
$= -5$

Substitute: $n = 2$
$S_2 = 3(2)^2 - 8(2)$
$= -4$

Substitute: $n = 3$
$S_3 = 3(3)^2 - 8(3)$
$= 3$

Substitute: $n = 4$
$S_4 = 3(4)^2 - 8(4)$
$= 16$

$t_1 = S_1$
$= -5$

$t_2 = S_2 - S_1$
$= 1$

$t_3 = S_3 - S_2$
$= 7$

$t_4 = S_4 - S_3$
$= 13$

17. Each number from 1 to 60 is written on one of 60 index cards. The cards are arranged in rows with equal lengths, and no cards are left over. The sum of the numbers in each row is 305. How many rows are there?

Determine the sum of the first 60 natural numbers:
$1 + 2 + 3 + \ldots + 60$
This is an arithmetic series with $t_1 = 1, d = 1$, and $t_{60} = 60$.
Use: $S_n = \dfrac{n(t_1 + t_n)}{2}$ Substitute: $n = 60, t_1 = 1, t_n = 60$
$S_n = \dfrac{60(1 + 60)}{2}$
$S_n = 1830$
The sum of the numbers in each row is 305, so the number of rows is:
$\dfrac{1830}{305} = 6$

18. Determine the arithmetic series that has these partial sums: $S_4 = 26$, $S_5 = 40$, and $S_6 = 57$

$t_6 = S_6 - S_5$
$t_6 = 57 - 40$
$= 17$

$t_5 = S_5 - S_4$
$t_5 = 40 - 26$
$= 14$

$d = t_6 - t_5$
$d = 17 - 14$
$= 3$

t_4 is: $t_5 - 3 = 11$

t_3 is: $t_4 - 3 = 8$

t_2 is: $t_3 - 3 = 5$

t_1 is: $t_2 - 3 = 2$

The arithmetic series is: $2 + 5 + 8 + 11 + 14 + 17 + \ldots$