Lesson 1.4 Exercises, pages 48–53

A

3. Write a geometric series for each geometric sequence.

a) 1, 4, 16, 64, 256, . . .

1 + 4 + 16 + 64 + 256 + . . .

b) 20, −10, 5, −2.5, 1.25, . . .

20 − 10 + 5 − 2.5 + 1.25 − . . .

4. Which series appear to be geometric? If the series could be geometric, determine S_5.

a) 2 + 4 + 8 + 16 + 32 + . . .

The series could be geometric.
S_5 is: 2 + 4 + 8 + 16 + 32 = 62

b) 2 − 4 + 8 − 16 + 32 − . . .

The series could be geometric.
S_5 is: 2 − 4 + 8 − 16 + 32 = 22

c) 1 + 4 + 9 + 16 + 25 + . . .

The series is not geometric.

d) −3 + 9 − 27 + 81 − 243 + . . .

The series could be geometric.
S_5 is: −3 + 9 − 27 + 81 − 243 = −183

5. Use the given data about each geometric series to determine the indicated value.

a) $t_1 = 1, r = 0.3$; determine S_8

Use: $S_n = \frac{t_1(1 - r^n)}{1 - r}, r \neq 1$

Substitute:

$n = 8, t_1 = 1, r = 0.3$

$S_8 = \frac{1(1 - 0.3^8)}{1 - 0.3}$

$S_8 \doteq 1.428$

b) $t_1 = \frac{3}{4}, r = \frac{1}{2}$; determine S_4

Use: $S_n = \frac{t_1(1 - r^n)}{1 - r}, r \neq 1$

Substitute:

$n = 4, t_1 = \frac{3}{4}, r = \frac{1}{2}$

$S_4 = \frac{\frac{3}{4}\left(1 - \left(\frac{1}{2}\right)^4\right)}{1 - \frac{1}{2}}$

$S_4 = \frac{45}{32}$, or approximately 1.406

B

6. Determine S_6 for each geometric series.

a) $2 + 10 + 50 + \ldots$

$t_1 = 2$ and r is: $\frac{10}{2} = 5$

Use: $S_n = \frac{t_1(1 - r^n)}{1 - r}, r \neq 1$

Substitute:

$n = 6, t_1 = 2, r = 5$

$S_6 = \frac{2(1 - 5^6)}{1 - 5}$

$S_6 = 7812$

b) $80 - 40 + 20 - \ldots$

$t_1 = 80$ and r is: $\frac{-40}{80} = -0.5$

Use: $S_n = \frac{t_1(1 - r^n)}{1 - r}, r \neq 1$

Substitute:

$n = 6, t_1 = 80, r = -0.5$

$S_6 = \frac{80(1 - (-0.5)^6)}{1 - (-0.5)}$

$S_6 = 52.5$

7. Determine S_{10} for each geometric series. Give the answers to 3 decimal places.

a) $0.1 + 0.01 + 0.001 + 0.0001 + \ldots$

$t_1 = 0.1$ and r is: $\frac{0.01}{0.1} = 0.1$

Use: $S_n = \frac{t_1(1 - r^n)}{1 - r}, r \neq 1$

Substitute:

$n = 10, t_1 = 0.1, r = 0.1$

$S_{10} = \frac{0.1(1 - 0.1^{10})}{1 - 0.1}$

$S_{10} \doteq 0.111$

b) $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \ldots$

$t_1 = 1$ and r is: $\frac{-\frac{1}{3}}{1} = -\frac{1}{3}$

Use: $S_n = \frac{t_1(1 - r^n)}{1 - r}, r \neq 1$

Substitute:

$n = 10, t_1 = 1, r = -\frac{1}{3}$

$S_{10} = \frac{1\left(1 - \left(-\frac{1}{3}\right)^{10}\right)}{1 - \left(-\frac{1}{3}\right)}$

$S_{10} \doteq 0.750$

8. a) Explain why this series appears to be geometric:

$1 + 5 + 25 + 125 + \ldots$

After the 1st term, each term is 5 times as great as the preceding term.

b) What information do you need to be certain that this is a geometric series?

I need to know that the series has a common ratio of 5.

c) What assumptions do you make when you identify or extend a geometric series?

I assume that the ratio of consecutive terms is the common ratio.

9. For each geometric series, determine how many terms it has then calculate its sum.

a) $1 - 2 + 4 - 8 + \ldots - 512$

$t_1 = 1$ and r is $\frac{-2}{1} = -2$

To determine n, use: $t_n = t_1r^{n-1}$

Substitute: $t_n = -512$, $t_1 = 1$, $r = -2$

$-512 = 1(-2)^{n-1}$

$(-2)^9 = (-2)^{n-1}$

$9 = n - 1$

$n = 10$

There are 10 terms.

To determine the sum, use: $S_n = \frac{t_1(1 - r^n)}{1 - r}, r \neq 1$

Substitute: $n = 10$, $t_1 = 1$, $r = -2$

$$S_{10} = \frac{1(1 - (-2)^{10})}{1 - (-2)}$$

$S_{10} = -341$

The sum is -341.

b) $-6561 + 2187 - 729 + 243 - \ldots - 1$

$t_1 = -6561$ and r is $\frac{2187}{-6561} = -\frac{1}{3}$

To determine n, use: $t_n = t_1r^{n-1}$

Substitute: $t_n = -1$, $t_1 = -6561$, $r = -\frac{1}{3}$

$$-1 = -6561\left(-\frac{1}{3}\right)^{n-1}$$

$$\frac{1}{6561} = \left(-\frac{1}{3}\right)^{n-1}$$

$$\left(-\frac{1}{3}\right)^{8} = \left(-\frac{1}{3}\right)^{n-1}$$

$8 = n - 1$

$n = 9$

There are 9 terms.

Use: $S_n = \frac{t_1(1 - r^n)}{1 - r}, r \neq 1$

Substitute: $n = 9$, $t_1 = -6561$, $r = -\frac{1}{3}$

$$S_9 = \frac{-6561\left(1 - \left(-\frac{1}{3}\right)^9\right)}{1 - \left(-\frac{1}{3}\right)}$$

$S_9 = -4921$

The sum is -4921.

10. Identify the terms in each partial sum of a geometric series.

a) $S_5 = 62, r = 2$

To determine t_1,

use $S_n = \frac{t_1(1 - r^n)}{1 - r}, r \neq 1$

Substitute: $n = 5, S_n = 62, r = 2$

$62 = \frac{t_1(1 - 2^5)}{1 - 2}$

$62 = 31t_1$

$t_1 = 2$

So, t_2 is $2(2) = 4$; t_3 is $4(2) = 8$; t_4 is $8(2) = 16$; t_5 is $16(2) = 32$

b) $S_8 = 1111.1111; r = 0.1$

To determine t_1,

use $S_n = \frac{t_1(1 - r^n)}{1 - r}, r \neq 1$

Substitute: $n = 8$,

$S_n = 1111.1111, r = 0.1$

$1111.1111 = \frac{t_1(1 - 0.1^8)}{1 - 0.1}$

$1111.1111 = 1.111\,111\,1t_1$

$t_1 = 1000$

So, t_2 is $1000(0.1) = 100$;

t_3 is $100(0.1) = 10$;

t_4 is $10(0.1) = 1$;

t_5 is $1(0.1) = 0.1$;

t_6 is $(0.1)(0.1) = 0.01$;

t_7 is $0.01(0.1) = 0.001$;

t_8 is $0.001(0.1) = 0.0001$

11. On Monday, Ian had 3 friends visit his personal profile on a social networking website. On Tuesday, each of these 3 friends had 3 different friends visit Ian's profile. On Wednesday, each of the 9 friends on Tuesday had 3 different friends visit Ian's profile.

a) Write the total number of friends who visited Ian's profile as a geometric series. What is the first term? What is the common ratio?

The 1st term is 3, the number of friends on Monday.

The common ratio is 3.

So, the geometric series is: $3 + 9 + 27$

b) Suppose this pattern continued for 1 week. What is the total number of people who visited Ian's profile? How do you know your answer is correct?

The geometric series continues and has 7 terms; one for each day of the week.

The series is: $3 + 9 + 27 + 81 + 243 + 729 + 2187$

Use: $S_n = \frac{t_1(1 - r^n)}{1 - r}, r \neq 1$ Substitute: $n = 7, t_1 = 3, r = 3$

$S_7 = \frac{3(1 - 3^7)}{1 - 3}$

$S_7 = 3279$

I checked my answer by using a calculator to add the seven terms.

12. Each stroke of a vacuum pump extracts 5% of the air in a 50-m^3 tank. How much air is removed after 50 strokes? Give the answer to the nearest tenth of a cubic metre.

Number of strokes	Volume removed	Volume remaining
1	$50(0.05) = 2.5$	$50(0.95) = 47.5$
2	$47.5(0.05) = 2.375$, or $2.5(0.95)$	$47.5(0.95) = 45.125$
3	$45.125(0.05) = 2.256\ 25$, or $2.5(0.95)^2$	

The volumes removed form a geometric series with 1st term 2.5 and common ratio 0.95.

Use: $S_n = \dfrac{t_1(1 - r^n)}{1 - r}, r \neq 1$ Substitute: $n = 50$, $t_1 = 2.5$, $r = 0.95$

$S_{50} = \dfrac{2.5(1 - 0.95^{50})}{1 - 0.95}$

$S_{50} = 46.1527\ldots$

After 50 strokes, about 46.2 m^3 of air is removed.

13. The sum of the first 10 terms of a geometric series is 1705. The common ratio is −2. Determine S_{11}. Explain your reasoning.

To determine t_1, use: $S_n = \dfrac{t_1(1 - r^n)}{1 - r}, r \neq 1$

Substitute: $S_n = 1705$, $n = 10$, $r = -2$

$1705 = \dfrac{t_1(1 - (-2)^{10})}{1 - (-2)}$

$1705 = -341t_1$

$t_1 = -5$

Then, $S_{11} = S_{10} + t_{11}$

$S_{11} = 1705 + (-5)(-2)^{10}$

$S_{11} = -3415$

C

14. Binary notation is used to represent numbers on a computer. For example, the number 1111 in base two represents $1(2)^3 + 1(2)^2 + 1(2)^1 + 1$, or 15 in base ten.

a) Why is the sum above an example of a geometric series?

Each term is one-half of the preceding term.

b) Which number in base ten is represented by 11 111 111 111 111 111 111 in base two? Explain your reasoning.

There are twenty 1s digits in the number,
so it can be written as the geometric series:
$1(2)^{19} + 1(2)^{18} + 1(2)^{17} + \ldots + 1(2)^{1} + 1$
This series has 20 terms, with 1st term 2^{19}
and common ratio 0.5.

Use: $S_n = \frac{t_1(1 - r^n)}{1 - r}, r \neq 1$ Substitute: $n = 20, t_1 = 2^{19}, r = 0.5$

$S_{20} = \frac{2^{19}(1 - 0.5^{20})}{1 - 0.5}$

$S_{20} = 1\ 048\ 575$

The number is 1 048 575.

15. Show how you can use geometric series to determine this sum:
$1 + 2 + 3 + 4 + 8 + 9 + 16 + 27 + 32 + 64 + 81 + 128 + 243 + 256 + 512$

This sum comprises two geometric series:
$1 + 3 + 9 + 27 + 81 + 243$ and
$2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 512$

For the first series

Use: $S_n = \frac{t_1(1 - r^n)}{1 - r}, r \neq 1$

Substitute: $n = 6, t_1 = 1, r = 3$

$S_6 = \frac{1(1 - 3^6)}{1 - 3}$

$S_6 = 364$

For the second series

Use: $S_n = \frac{t_1(1 - r^n)}{1 - r}, r \neq 1$

Substitute: $n = 9, t_1 = 2, r = 2$

$S_9 = \frac{2(1 - 2^9)}{1 - 2}$

$S_9 = 1022$

The sum is: $364 + 1022 = 1386$

16. Determine the common ratio of a geometric series that has these partial sums: $S_3 = -\frac{49}{8}, S_4 = -\frac{105}{16}, S_5 = -\frac{217}{32}$

$S_4 = S_3 + t_4$

Substitute for S_4 and S_3.

$-\frac{105}{16} = -\frac{49}{8} + t_4$

$t_4 = -\frac{7}{16}$

$S_5 = S_4 + t_5$

Substitute for S_5 and S_4.

$-\frac{217}{32} = -\frac{105}{16} + t_5$

$t_5 = -\frac{7}{32}$

$t_5 = t_4(r)$

$-\frac{7}{32} = -\frac{7}{16}(r)$

$r = \frac{1}{2}$

The common ratio is $\frac{1}{2}$.