CUMULATIVE REVIEW Chapters 1-2, pages 165–168

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1. a) **i)** Determine this sum:

$-5.5 - 2 + 1.5 + 5 + 8.5 + 12 + 15.5$

Use: $S_n = \frac{n(t_1 + t_n)}{2}$ Substitute: $n = 7, t_1 = -5.5, t_n = 15.5$

$S_7 = \frac{7(-5.5 + 15.5)}{2}$

$S_7 = 35$

ii) For the arithmetic series in part i, determine the value of t_{18}.

Use: $t_n = t_1 + d(n - 1)$ Substitute: $n = 18, t_1 = -5.5, d = 3.5$

$t_{18} = -5.5 + 3.5(18 - 1)$

$t_{18} = 54$

b) **i)** Determine the sum of the first 16 terms of this arithmetic series: $17 + 11 + 5 - 1 - 7 - 13 - \ldots$

Use: $S_n = \frac{n[2t_1 + d(n - 1)]}{2}$ Substitute: $n = 16, t_1 = 17, d = -6$

$S_{16} = \frac{16[2(17) - 6(16 - 1)]}{2}$

$S_{16} = -448$

ii) For the series in part i, which term has the value -121?

Use: $t_n = t_1 + d(n - 1)$ Substitute: $t_n = -121, t_1 = 17, d = -6$

$-121 = 17 - 6(n - 1)$

$138 = 6(n - 1)$

$23 = n - 1$

$n = 24$

t_{24} has the value -121.

2. Write the first 5 terms of each geometric sequence.

a) Each term is a power of 3.

Sample response: Choose t_1 that is a power of 3, such as 9.
Multiply each term by a common ratio that is a power of 3, such as 27.
One sequence is: 9, 243, 6561, 177 147, 4 782 969, . . .

b) Each term is a negative integer.

Sample response: Choose t_1 that is a negative integer, such as -4.
Multiply each term by a common ratio that is a positive integer, such as 5.
One sequence is: $-4, -20, -100, -500, -2500, \ldots$

c) $t_5 = 32$

Sample response: Work backward.
Divide 32 by a power of 2, such as 0.5.
Then divide successive terms by 0.5.
32, 64, 128, 256, 512
One sequence is: 512, 256, 128, 64, 32, . . .

3. From 2000 to 2008, the amount of electricity supplied by wind energy in Canada increased by about 40% annually. Wind energy in 2008 produced enough electricity to supply about 716 000 homes. To the nearest thousand, about how many homes might be supplied by wind energy in 2018? Explain your reasoning and identify any assumptions you made.

I assume that the electricity supplied by wind energy continues to increase by 40% each year.
The annual numbers of homes supplied by wind energy form a geometric sequence with $t_1 = 716\,000$ and $r = 1.4$.
The number of homes in 2008 is t_1, so the number of homes in 2018 is t_{11}.
Use: $t_n = t_1 r^{n-1}$ Substitute: $n = 11$, $t_1 = 716\,000$, $r = 1.4$

$$t_{11} = 716\,000(1.4)^{11-1}$$
$$= 20\,710\,633.3...$$

About 20 711 000 homes will be supplied in 2018.

4. Identify whether each infinite geometric series converges. Determine the sum of each series that converges.

a) $0.7 - 0.07 + 0.007 - 0.0007 + \ldots$

r is: $\frac{-0.07}{0.7} = -0.1$
The common ratio is between -1 and 1, so the series converges.
In $S_\infty = \frac{t_1}{1 - r}$, substitute: $t_1 = 0.7$, $r = -0.1$

$$S_\infty = \frac{0.7}{1 - (-0.1)}$$
$$S_\infty = \frac{7}{11}, \text{ or } 0.\overline{63}$$

b) $-4 - 3 - \frac{9}{4} - \frac{27}{16} - \ldots$

r is: $\frac{-3}{-4} = 0.75$
The common ratio is between -1 and 1, so the series converges.
In $S_\infty = \frac{t_1}{1 - r}$, substitute: $t_1 = -4$, $r = 0.75$

$$S_\infty = \frac{-4}{1 - 0.75}$$
$$S_\infty = -16$$

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5. Evaluate.

a) $\dfrac{|(-2) - (-8)|}{|(-9) + (-3)|}$

$= \dfrac{|6|}{|-12|}$

$= \dfrac{1}{2}$

b) $\frac{1}{2}\left|1\frac{1}{4} - 2\frac{1}{8}\right| - \left|-9\frac{1}{3} + 7\frac{2}{3}\right|$

$= \frac{1}{2}\left|-\frac{7}{8}\right| - \left|-1\frac{2}{3}\right|$

$= \frac{7}{16} - \frac{5}{3}$

$= -\frac{59}{48}$, or $-1\frac{11}{48}$

6. Simplify each radical, and state the values of the variables for which the radical is defined.

a) $\sqrt{63x^3}$

$= \sqrt{9 \cdot 7} \cdot \sqrt{x^2} \cdot \sqrt{x}$

$= 3x\sqrt{7x}, x \geq 0$

b) $\sqrt[3]{-54y^7}$

$= \sqrt[3]{-27 \cdot 2} \cdot \sqrt[3]{y^6} \cdot \sqrt[3]{y}$

$= -3y^2\sqrt[3]{2y}, y \in \mathbb{R}$

7. Simplify.

a) $2n\sqrt{5m} - n\sqrt[3]{4m} - 4n\sqrt{5m} + 5n\sqrt[3]{4m}, m \geq 0, n \in \mathbb{R}$

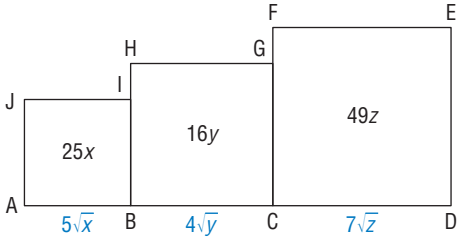
$= -2n\sqrt{5m} + 4n\sqrt[3]{4m}$

b) $4\sqrt{v^7w^2} - 3\sqrt{v^4w^5} - 6\sqrt{v^7w^2} + 7\sqrt{v^4w^5}, v, w \geq 0$

$= -2\sqrt{v^7w^2} + 4\sqrt{v^4w^5}$

$= -2v^3w\sqrt{v} + 4v^2w^2\sqrt{w}$

8. Three squares, with areas shown below, have adjacent sides touching. Determine the perimeter of the shape they form.



The side length of each square is the square root of its area.
The perimeter is:
AB + BC + CD + DE + EF + FG + GH + HI + IJ + JA
$= 3AB + 2BC + 3CD + FG + HI$
$= 3(5\sqrt{x}) + 2(4\sqrt{y}) + 3(7\sqrt{z}) + (7\sqrt{z} - 4\sqrt{y}) + (4\sqrt{y} - 5\sqrt{x})$
$= 15\sqrt{x} + 8\sqrt{y} + 21\sqrt{z} + 7\sqrt{z} - 5\sqrt{x}$
$= 10\sqrt{x} + 8\sqrt{y} + 28\sqrt{z}$

9. Simplify.

a) $(6 + \sqrt{3})(4 - \sqrt{3}) - \sqrt{3}(2 - \sqrt{3})$

$= 24 - 6\sqrt{3} + 4\sqrt{3} - 3 - 2\sqrt{3} + 3$

$= 24 - 4\sqrt{3}$

b) $(\sqrt{m} + 3\sqrt{n})^2 - (2\sqrt{m} - 5\sqrt{n})(2\sqrt{m} + 5\sqrt{n})$

$= m + 3\sqrt{mn} + 3\sqrt{mn} + 9n - 4m + 25n$

$= -3m + 6\sqrt{mn} + 34n;\ m, n \geq 0$

c) $\dfrac{15\sqrt{2} + 5\sqrt{3}}{\sqrt{20}}$

$= \dfrac{15\sqrt{2} + 5\sqrt{3}}{2\sqrt{5}}$

$= \dfrac{(15\sqrt{2} + 5\sqrt{3})}{2\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}}$

$= \dfrac{15\sqrt{10} + 5\sqrt{15}}{10}$

$= \dfrac{3\sqrt{10} + \sqrt{15}}{2}$

d) $\dfrac{4\sqrt{7} - 3\sqrt{2}}{5\sqrt{2} + 2\sqrt{7}}$

$= \dfrac{(4\sqrt{7} - 3\sqrt{2})}{(5\sqrt{2} + 2\sqrt{7})} \cdot \dfrac{(5\sqrt{2} - 2\sqrt{7})}{(5\sqrt{2} - 2\sqrt{7})}$

$= \dfrac{20\sqrt{14} - 56 - 30 + 6\sqrt{14}}{50 - 28}$

$= \dfrac{26\sqrt{14} - 86}{22}$

$= \dfrac{13\sqrt{14} - 43}{11}$

10. Solve each equation. Verify the solution.

a) $4 - 5\sqrt{6x} = -5 - 4\sqrt{6x}$

$x \geq 0$

$9 = \sqrt{6x}$

$81 = 6x$

$x = 13.5$

Verify. L.S. = R.S., so the solution is: $x = 13.5$

b) $\sqrt{x} + 14 = 3\sqrt{x} + 10$

$x \geq 0$

$2\sqrt{x} = 4$

$\sqrt{x} = 2$

$x = 4$

Verify. L.S. = R.S., so the root is: $x = 4$