REVIEW, pages 156–161

2.1

1. For each pair of numbers, write two expressions to represent the distance between the numbers on a number line, then determine this distance.

a) $1\frac{3}{8}$ and $3\frac{1}{4}$

$\left|1\frac{3}{8} - 3\frac{1}{4}\right|$ and $\left|3\frac{1}{4} - 1\frac{3}{8}\right|$

$$\begin{aligned}\left|1\frac{3}{8} - 3\frac{1}{4}\right| &= \left|\frac{11}{8} - \frac{13}{4}\right|\\ &= \left|\frac{11}{8} - \frac{26}{8}\right|\\ &= \left|-\frac{15}{8}\right|\\ &= \frac{15}{8}, \text{ or } 1\frac{7}{8}\end{aligned}$$

The numbers are $1\frac{7}{8}$ units apart on a number line.

b) 7.5 and −3.75

$|7.5 - (-3.75)|$, or $|7.5 + 3.75|$, and $|-3.75 - 7.5|$

$$\begin{aligned}|7.5 + 3.75| &= |11.25|\\ &= 11.25\end{aligned}$$

The numbers are 11.25 units apart on a number line.

2.2

2. Arrange in order from least to greatest.

a) $3\sqrt{6}, \sqrt{24}, -2\sqrt{6}, \sqrt{96}$

Each radical has index 2. Write each mixed radical as an entire radical.

$$\begin{aligned}3\sqrt{6} &= \sqrt{3^2}\cdot\sqrt{6}\\ &= \sqrt{9\cdot 6}\\ &= \sqrt{54}\end{aligned} \qquad \sqrt{24} \qquad \begin{aligned}-2\sqrt{6} &= -\sqrt{2^2}\cdot\sqrt{6}\\ &= -\sqrt{4\cdot 6}\\ &= -\sqrt{24}\end{aligned} \qquad \sqrt{96}$$

$-\sqrt{24}$ is negative so it has the least value.

Compare the radicands of the other radicals: $24 < 54 < 96$

So, from least to greatest: $-2\sqrt{6}, \sqrt{24}, 3\sqrt{6}, \sqrt{96}$

b) $\frac{5}{8}, \sqrt{\frac{72}{50}}, 2\sqrt{\frac{1}{16}}, \frac{\sqrt{9}}{5}$

Each radical has index 2.

Simplify each radical.

$$\frac{5}{8} \qquad \begin{aligned}\sqrt{\frac{72}{50}} &= \sqrt{\frac{36\cdot 2}{25\cdot 2}}\\ &= \sqrt{\frac{36}{25}}\\ &= \frac{6}{5}\end{aligned} \qquad \begin{aligned}2\sqrt{\frac{1}{16}} &= 2\left(\frac{1}{4}\right)\\ &= \frac{1}{2}\end{aligned} \qquad \frac{\sqrt{9}}{5} = \frac{3}{5}$$

Compare the fractions: $\frac{1}{2} < \frac{3}{5} < \frac{5}{8} < \frac{6}{5}$

So, from least to greatest: $2\sqrt{\frac{1}{16}}, \frac{\sqrt{9}}{5}, \frac{5}{8}, \sqrt{\frac{72}{50}}$

3. Write each entire radical as a mixed radical, if possible.

a) $\sqrt[3]{-\frac{48}{250}} = \sqrt[3]{\frac{-48}{250}}$
$= \sqrt[3]{\frac{-8 \cdot 6}{125 \cdot 2}}$
$= \frac{-2}{5}\sqrt[3]{\frac{6}{2}}$
$= -\frac{2}{5}\sqrt[3]{3}$

b) $\sqrt[4]{\frac{32}{243}} = \sqrt[4]{\frac{16 \cdot 2}{81 \cdot 3}}$
$= \frac{2}{3}\sqrt[4]{\frac{2}{3}}$

4. Write the values of the variable for which each radical is defined, then simplify the radical, if possible.

a) $\sqrt{16x}$

$\sqrt{16x} \in \mathbb{R}$ when
$16x \geq 0$; that is,
when $x \geq 0$.
$\sqrt{16x} = \sqrt{16 \cdot x}$
$= 4\sqrt{x}$

b) $\sqrt{64x^2}$

$\sqrt{64x^2} \in \mathbb{R}$ when
$64x^2 \geq 0$.
$64 > 0$ and $x^2 \geq 0$
So, $\sqrt{64x^2}$ is defined
for $x \in \mathbb{R}$.
$\sqrt{64x^2} = \sqrt{64 \cdot x^2}$
$= 8|x|$

c) $\sqrt[3]{-64x^3}$

Since the cube root of
a number is defined
for all real values
of x, the radical is
defined for $x \in \mathbb{R}$.
$\sqrt[3]{-64x^3}$
$= \sqrt[3]{-64 \cdot x^3}$
$= -4x$

d) $\sqrt[4]{16x^6}$

$\sqrt[4]{16x^6} \in \mathbb{R}$ when
$16x^6 \geq 0$.
$16 > 0$ and $x^6 \geq 0$
So, $\sqrt[4]{16x^6}$
is defined for $x \geq 0$.
$\sqrt[4]{16x^6} = \sqrt[4]{16 \cdot x^4 \cdot x^2}$
$= 2|x|\sqrt[4]{x^2}$

2.3

5. Identify the values of the variables for which each radical is defined where necessary, then simplify.

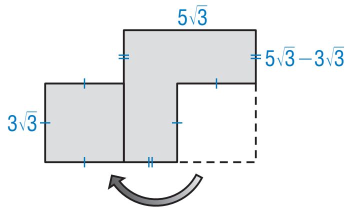
a) $\sqrt{72} + \sqrt{50} - \sqrt{18}$

$= \sqrt{36 \cdot 2} + \sqrt{25 \cdot 2} - \sqrt{9 \cdot 2}$
$= 6\sqrt{2} + 5\sqrt{2} - 3\sqrt{2}$
$= 8\sqrt{2}$

b) $\sqrt[3]{16x} - \sqrt[3]{375x} + 3\sqrt[3]{2x}$

The cube root of a number is defined
for all real numbers. So, each radical
is defined for $x \in \mathbb{R}$.
$= \sqrt[3]{8 \cdot 2 \cdot x} - \sqrt[3]{125 \cdot 3 \cdot x} + 3\sqrt[3]{2x}$
$= 2\sqrt[3]{2x} - 5\sqrt[3]{3x} + 3\sqrt[3]{2x}$
$= 5\sqrt[3]{2x} - 5\sqrt[3]{3x}$

6. A square with area 75 square units has a square corner of area 27 square units moved as shown. Determine the perimeter of the resulting shape. Describe the steps you took to solve the problem.



The side length of a square is the square root of its area.
So, the side length of the square with area 75 square units is:
$\sqrt{75} = 5\sqrt{3}$ units
The side length of the square with area 27 square units is:
$\sqrt{27} = 3\sqrt{3}$ units
Label the diagram.
Perimeter of shape formed $= 5(3\sqrt{3}) + 5\sqrt{3} + 3(5\sqrt{3} - 3\sqrt{3})$
$= 15\sqrt{3} + 5\sqrt{3} + 3(2\sqrt{3})$
$= 26\sqrt{3}$
The perimeter of the shape formed is $26\sqrt{3}$ units.

2.4

7. Identify the values of the variable for which each expression is defined where necessary, then expand and simplify.

a) $(\sqrt{5} - \sqrt{7})(\sqrt{5} + \sqrt{7})$

$= \sqrt{5}(\sqrt{5} + \sqrt{7}) - \sqrt{7}(\sqrt{5} + \sqrt{7})$
$= 5 + \sqrt{35} - \sqrt{35} - 7$
$= -2$

b) $(2\sqrt{a} + \sqrt{b})(\sqrt{a} + \sqrt{b})$

The radicands cannot be negative, so $a \geq 0$ and $b \geq 0$.
$(2\sqrt{a} + \sqrt{b})(\sqrt{a} + \sqrt{b})$
$= 2\sqrt{a}(\sqrt{a} + \sqrt{b}) + \sqrt{b}(\sqrt{a} + \sqrt{b})$
$= 2a + 2\sqrt{ab} + \sqrt{ab} + b$
$= 2a + 3\sqrt{ab} + b$

8. Rationalize the denominator.

a) $\dfrac{3\sqrt{5} - \sqrt{7}}{5\sqrt{3}}$

$= \dfrac{(3\sqrt{5} - \sqrt{7})}{5\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}$

$= \dfrac{3\sqrt{5} \cdot \sqrt{3} - \sqrt{7} \cdot \sqrt{3}}{5\sqrt{3} \cdot \sqrt{3}}$

$= \dfrac{3\sqrt{15} - \sqrt{21}}{15}$

b) $\dfrac{3\sqrt{2} + 4\sqrt{3}}{\sqrt{8}} = \dfrac{3\sqrt{2} + 4\sqrt{3}}{2\sqrt{2}}$

$= \dfrac{3\sqrt{2} + 4\sqrt{3}}{2\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{3\sqrt{2} \cdot \sqrt{2} + 4\sqrt{3} \cdot \sqrt{2}}{2\sqrt{2} \cdot \sqrt{2}}$

$= \dfrac{6 + 4\sqrt{6}}{4}$

$= \dfrac{3 + 2\sqrt{6}}{2}$

9. Simplify.

a) $\dfrac{2\sqrt{6}}{\sqrt{7} + \sqrt{5}}$

$= \dfrac{2\sqrt{6}}{(\sqrt{7} + \sqrt{5})} \cdot \dfrac{(\sqrt{7} - \sqrt{5})}{(\sqrt{7} - \sqrt{5})}$

$= \dfrac{2\sqrt{6}(\sqrt{7}) - 2\sqrt{6}(\sqrt{5})}{(\sqrt{7})^2 - (\sqrt{5})^2}$

$= \dfrac{2\sqrt{42} - 2\sqrt{30}}{2}$

$= \sqrt{42} - \sqrt{30}$

b) $\dfrac{3\sqrt{5} - 4\sqrt{3}}{6\sqrt{2} - \sqrt{3}}$

$= \dfrac{(3\sqrt{5} - 4\sqrt{3})}{(6\sqrt{2} - \sqrt{3})} \cdot \dfrac{(6\sqrt{2} + \sqrt{3})}{(6\sqrt{2} + \sqrt{3})}$

$= \dfrac{3\sqrt{5}(6\sqrt{2} + \sqrt{3}) - 4\sqrt{3}(6\sqrt{2} + \sqrt{3})}{(6\sqrt{2})^2 - (\sqrt{3})^2}$

$= \dfrac{18\sqrt{10} + 3\sqrt{15} - 24\sqrt{6} - 12}{72 - 3}$

$= \dfrac{3(6\sqrt{10} + \sqrt{15} - 8\sqrt{6} - 4)}{69}$

$= \dfrac{6\sqrt{10} + \sqrt{15} - 8\sqrt{6} - 4}{23}$

10. Identify the values of the variable for which each expression is defined, then expand and simplify.

a) $2\sqrt{a}(3\sqrt{b} + \sqrt{a})^2$

The radicands cannot be negative, so $a \geq 0$ and $b \geq 0$.

$2\sqrt{a}(3\sqrt{b} + \sqrt{a})^2 = 2\sqrt{a}(3\sqrt{b} + \sqrt{a})(3\sqrt{b} + \sqrt{a})$

$= 2\sqrt{a}\left[(3\sqrt{b})(3\sqrt{b} + \sqrt{a}) + \sqrt{a}(3\sqrt{b} + \sqrt{a})\right]$

$= 2\sqrt{a}[9b + 3\sqrt{ab} + 3\sqrt{ab} + a]$

$= 2\sqrt{a}[9b + 6\sqrt{ab} + a]$

$= 18b\sqrt{a} + 12\sqrt{a^2b} + 2a\sqrt{a}$

$= 18b\sqrt{a} + 12a\sqrt{b} + 2a\sqrt{a}$

b) $(3\sqrt{x} + 2\sqrt{y})^2 - (3\sqrt{x} - 2\sqrt{y})^2$

The radicands cannot be negative, so $x \geq 0$ and $y \geq 0$.

$(3\sqrt{x} + 2\sqrt{y})^2 - (3\sqrt{x} - 2\sqrt{y})^2$

$= (3\sqrt{x} + 2\sqrt{y})(3\sqrt{x} + 2\sqrt{y}) - (3\sqrt{x} - 2\sqrt{y})(3\sqrt{x} - 2\sqrt{y})$

$= 3\sqrt{x}(3\sqrt{x} + 2\sqrt{y}) + 2\sqrt{y}(3\sqrt{x} + 2\sqrt{y})$

$\quad - \left[3\sqrt{x}(3\sqrt{x} - 2\sqrt{y}) - 2\sqrt{y}(3\sqrt{x} - 2\sqrt{y})\right]$

$= 9x + 6\sqrt{xy} + 6\sqrt{xy} + 4y - [9x - 6\sqrt{xy} - 6\sqrt{xy} + 4y]$

$= 9x + 12\sqrt{xy} + 4y - 9x + 12\sqrt{xy} - 4y$

$= 24\sqrt{xy}$

2.5

11. Determine the root of each equation. Verify the solution.

a) $5 = \sqrt{2x + 7}$

$2x + 7 \geq 0$; that is, $x \geq -\frac{7}{2}$

$5 = \sqrt{2x + 7}$

$5^2 = (\sqrt{2x + 7})^2$

$25 = 2x + 7$

$2x = 18$

$x = 9$

b) $1 - 2\sqrt{3x} = 4 - 3\sqrt{3x}$

$3x \geq 0$; that is, $x \geq 0$

$1 - 2\sqrt{3x} = 4 - 3\sqrt{3x}$

$\sqrt{3x} = 3$

$(\sqrt{3x})^2 = 3^2$

$3x = 9$

$x = 3$

12. Which equations have real roots? Justify your answers.

a) $2\sqrt{x + 5} = 3\sqrt{5x - 11}$

$x + 5 \geq 0$; that is, $x \geq -5$

$5x - 11 \geq 0$; that is, $x \geq \frac{11}{5}$

So, for both radicals to be defined, $x \geq \frac{11}{5}$

$2\sqrt{x + 5} = 3\sqrt{5x - 11}$

$(2\sqrt{x + 5})^2 = (3\sqrt{5x - 11})^2$

$4(x + 5) = 9(5x - 11)$

$4x + 20 = 45x - 99$

$119 = 41x$

$x = \frac{119}{41}$, or $2\frac{37}{41}$

Since $x = 2\frac{37}{41}$ lies in the set of possible values for x, the equation has a real root.

b) $\sqrt{11x + 2} + 8 = 3$

$11x + 2 \geq 0$; that is, $x \geq -\frac{2}{11}$

$\sqrt{11x + 2} + 8 = 3$

$\sqrt{11x + 2} = -5$

The left side of the equation is greater than or equal to 0.

The right side of the equation is negative, -5.

So, no real solutions are possible.

The equation has no real roots.

13. The approximate speed at which a tsunami can travel is given by the formula $S = \sqrt{9.8d}$, where S is the speed of the tsunami in metres per second, and d is the mean depth of the water in metres.
A tsunami is travelling at 36 m/s. What is the mean depth of the water to the nearest metre?

$S = \sqrt{9.8d}$ Substitute: $S = 36$

$36 = \sqrt{9.8d}$

$(36)^2 = (\sqrt{9.8d})^2$

$1296 = 9.8d$

$\frac{1296}{9.8} = d$

$d = 132.2448\ldots$

The mean depth of the water is about 132 m.