

Lesson 3.1 Exercises, pages 176–182

A

3. Factor.

a) $x^2 - 100$

$$= (x)^2 - (10)^2$$

$$= (x + 10)(x - 10)$$

b) $49x^2 - 1$

$$= (7x)^2 - (1)^2$$

$$= (7x + 1)(7x - 1)$$

c) $4 - 49n^2$

$$= (2)^2 - (7n)^2$$

$$= (2 + 7n)(2 - 7n)$$

d) $64x^2 - 25y^2$

$$= (8x)^2 - (5y)^2$$

$$= (8x - 5y)(8x + 5y)$$

4. Factor.

a) $x^2 + 7x + 10$

Find 2 numbers with
sum 7 and product 10.

$$= (x + 2)(x + 5)$$

b) $x^2 - 11x + 30$

Find 2 numbers with sum
-11 and product 30.

$$= (x - 5)(x - 6)$$

c) $m^2 + 4m - 45$

Find 2 numbers with
sum 4 and product -45 .
 $= (m + 9)(m - 5)$

d) $y^2 - 2y - 63$

Find 2 numbers with sum
 -2 and product -63 .
 $= (y - 9)(y + 7)$

5. Factor.

a) $5x^2 + 9x + 4$

Guess and test factors of
5 with factors of 4.
 $= (5x + 4)(x + 1)$

b) $4x^2 - 7x + 3$

Guess and test
factors of 4 with
negative factors of 3.
 $= (x - 1)(4x - 3)$

c) $6x^2 + x - 2$

Guess and test factors of
6 with factors of -2 .
 $= (3x + 2)(2x - 1)$

d) $2x^2 - 9x + 4$

Guess and test factors of 2
with negative factors of 4.
 $= (x - 4)(2x - 1)$

6. Factor.

a) $9x^2 + 6x + 1$

Guess and test factors of
9 with factors of 1.
 $= (3x + 1)(3x + 1)$,
or $(3x + 1)^2$

b) $25x^2 + 20x + 4$

This is a perfect square
trinomial. It can be written
as: $(5x)^2 + 2(5x)(2) + 2^2$
 $= (5x + 2)^2$

c) $36x^2 - 60x + 25$

This is a perfect square
trinomial. It can be written
as: $(6x)^2 + 2(6x)(-5) + (-5)^2$
 $= (6x - 5)^2$

d) $4y^2 - 20y + 25$

This is a perfect square
trinomial. It can be written
as: $(2y)^2 + 2(2y)(-5) + (-5)^2$
 $= (2y - 5)^2$

B7. Determine whether $x + 5$ is a factor of each polynomial.

a) $2x^2 - 2x - 40$

Write the trinomial as:
 $(x + 5)(2x + b) =$
 $2x^2 + (b + 10)x + 5b$
Equate constant terms.
 $5b = -40$, so $b = -8$
Check:
 $(x + 5)(2x - 8) =$
 $2x^2 + 2x - 40$
So, $x + 5$ is not a factor.

b) $3x^2 + 13x - 10$

Write the trinomial as:
 $(x + 5)(3x + b) =$
 $3x^2 + (b + 15)x + 5b$
Equate constant terms.
 $5b = -10$, so $b = -2$
Check:
 $(x + 5)(3x - 2) =$
 $3x^2 + 13x - 10$
So, $x + 5$ is a factor.

8. Determine whether $2x + 1$ is a factor of each polynomial.

a) $6x^2 - 13x - 8$

Write the trinomial as:

$$(2x + 1)(3x + b) =$$

$$6x^2 + (2b + 3)x + b$$

Equate constant terms.

$$b = -8$$

Check:

$$(2x + 1)(3x - 8) =$$

$$6x^2 - 13x - 8$$

So, $2x + 1$ is a factor.

b) $2x^2 - 8$

Write the binomial as:

$$(2x + 1)(x + b) =$$

$$2x^2 + (2b + 1)x + b$$

Equate constant terms.

$$b = -8$$

Check:

$$(2x + 1)(x - 8) =$$

$$2x^2 - 15x - 8$$

So, $2x + 1$ is not a factor.

9. Factor.

a) $2x^2 - 50y^2$

$$= 2(x^2 - 25y^2)$$

$$= 2(x + 5y)(x - 5y)$$

b) $0.1x^2 - 0.001$

$$= 0.1(x^2 - 0.01)$$

$$= 0.1(x + 0.1)(x - 0.1)$$

c) $20x^2 - 125y^2$

$$= 5(4x^2 - 25y^2)$$

$$= 5(2x + 5y)(2x - 5y)$$

d) $\frac{1}{100}x^2 - \frac{1}{25}y^2$

$$= \frac{1}{25}\left(\frac{1}{4}x^2 - y^2\right)$$

$$= \frac{1}{25}\left(\frac{1}{2}x + y\right)\left(\frac{1}{2}x - y\right)$$

10. Factor.

a) $2x^2 + 16x + 24$

$$= 2(x^2 + 8x + 12)$$

Find 2 numbers with sum 8 and product 12.

$$= 2(x + 2)(x + 6)$$

b) $3x^2 - 9x - 30$

$$= 3(x^2 - 3x - 10)$$

Find 2 numbers with sum -3 and product -10 .

$$= 3(x - 5)(x + 2)$$

c) $x^2 + \frac{5}{2}x - 6$

$$= \frac{1}{2}(2x^2 + 5x - 12)$$

Guess and test factors of 2 with factors of -12 .

$$= \frac{1}{2}(2x - 3)(x + 4)$$

d) $x^2 + 2.5x - 1.5$

$$= 0.5(2x^2 + 5x - 3)$$

Guess and test factors of 2 with factors of -3 .

$$= 0.5(2x - 1)(x + 3)$$

11. Factor each polynomial.

a) $\frac{x^2}{9} - \frac{4}{25}$

$$\begin{aligned} &= \left(\frac{x}{3}\right)^2 - \left(\frac{2}{5}\right)^2 \\ &= \left(\frac{x}{3} + \frac{2}{5}\right)\left(\frac{x}{3} - \frac{2}{5}\right) \end{aligned}$$

b) $6 + 5x - x^2$

Guess and test factors of 6
with factors of -1 .
 $= (6 - x)(1 + x)$

c) $-x^2 + \frac{121}{64}$

$$\begin{aligned} &= \left(\frac{11}{8}\right)^2 - (x)^2 \\ &= \left(\frac{11}{8} + x\right)\left(\frac{11}{8} - x\right) \end{aligned}$$

d) $7 - \frac{5}{3}x - 2x^2$

$$\begin{aligned} &= \frac{1}{3}(21 - 5x - 6x^2) \\ &\text{Use decomposition.} \\ &= \frac{1}{3}(21 - 14x + 9x - 6x^2) \\ &= \frac{1}{3}[7(3 - 2x) + 3x(3 - 2x)] \\ &= \frac{1}{3}(3 - 2x)(7 + 3x) \end{aligned}$$

12. Factor each polynomial expression.

a) i) $9x^2 - 4y^2$

$$= (3x + 2y)(3x - 2y)$$

ii) $9(x - 3)^2 - 4(2y + 1)^2$

$$\begin{aligned} &= [3(x - 3)]^2 - [2(2y + 1)]^2 \\ &= [3(x - 3) + 2(2y + 1)][3(x - 3) - 2(2y + 1)] \\ &= [3x - 9 + 4y + 2][3x - 9 - 4y - 2] \\ &= (3x + 4y - 7)(3x - 4y - 11) \end{aligned}$$

b) i) $50x^2 - 162y^2$

$$\begin{aligned} &= 2(25x^2 - 81y^2) \\ &= 2(5x + 9y)(5x - 9y) \end{aligned}$$

ii) $50(2x - 5)^2 - 162(3y - 2)^2$

$$\begin{aligned} &= 2[25(2x - 5)^2 - 81(3y - 2)^2] \\ &= 2([5(2x - 5)]^2 - [9(3y - 2)]^2) \\ &= 2[5(2x - 5) + 9(3y - 2)][5(2x - 5) - 9(3y - 2)] \\ &= 2[10x - 25 + 27y - 18][10x - 25 - 27y + 18] \\ &= 2(10x + 27y - 43)(10x - 27y - 7) \end{aligned}$$

13. Factor each polynomial expression.

$$\begin{aligned}\text{a) } 16(2x - 7)^2 - 25(y + 2)^2 &= [4(2x - 7)]^2 - [5(y + 2)]^2 \\ &= [4(2x - 7) + 5(y + 2)][4(2x - 7) - 5(y + 2)] \\ &= (8x - 28 + 5y + 10)(8x - 28 - 5y - 10) \\ &= (8x + 5y - 18)(8x - 5y - 38)\end{aligned}$$

$$\begin{aligned}\text{b) } 121(x + 3)^2 - 36(2y - 5)^2 &= [11(x + 3)]^2 - [6(2y - 5)]^2 \\ &= [11(x + 3) + 6(2y - 5)][11(x + 3) - 6(2y - 5)] \\ &= (11x + 33 + 12y - 30)(11x + 33 - 12y + 30) \\ &= (11x + 12y + 3)(11x - 12y + 63)\end{aligned}$$

14. Use two different strategies to factor this polynomial expression: $(2x + 3)^2 - (x - 5)^2$. Which strategy is more efficient?

Strategy 1:

$$\begin{aligned}(2x + 3)^2 - (x - 5)^2 &= [(2x + 3) + (x - 5)][(2x + 3) - (x - 5)] \\ &= (2x + 3 + x - 5)(2x + 3 - x + 5) \\ &= (3x - 2)(x + 8)\end{aligned}$$

Strategy 2:

$$\begin{aligned}(2x + 3)^2 - (x - 5)^2 &= (2x + 3)(2x + 3) - (x - 5)(x - 5) \\ &= (4x^2 + 12x + 9) - (x^2 - 10x + 25) \\ &= 4x^2 + 12x + 9 - x^2 + 10x - 25 \\ &= 3x^2 + 22x - 16 \\ &= (3x - 2)(x + 8)\end{aligned}$$

Strategy 1 is more efficient.

15. Factor each polynomial expression.

$$\text{a) i) } 3x^2 + 19x + 16 \qquad \text{ii) } 3(2x - 1)^2 + 19(2x - 1) + 16$$

Guess and test factors of 3 with factors of 16.

$$= (3x + 16)(x + 1)$$

Use the factors in part i.

$$\begin{aligned}&= [3(2x - 1) + 16][(2x - 1) + 1] \\ &= [6x - 3 + 16][2x - 1 + 1] \\ &= (6x + 13)(2x)\end{aligned}$$

$$\text{b) i) } 12x^2 + 17x - 5 \qquad \text{ii) } 12(4x - 1)^2 + 17(4x - 1) - 5$$

Guess and test factors of 12 with factors of -5.

$$= (4x - 1)(3x + 5)$$

Use the factors in part i.

$$\begin{aligned}&= [4(4x - 1) - 1][3(4x - 1) + 5] \\ &= [16x - 4 - 1][12x - 3 + 5] \\ &= (16x - 5)(12x + 2) \\ &= 2(16x - 5)(6x + 1)\end{aligned}$$

16. Factor each polynomial expression.

a) $(7x - 5)^2 - 8(7x - 5) + 15$

Find 2 numbers with sum -8 and product 15 .

$$\begin{aligned} &= [(7x - 5) - 3][(7x - 5) - 5] \\ &= (7x - 5 - 3)(7x - 5 - 5) \\ &= (7x - 8)(7x - 10) \end{aligned}$$

b) $9(2x + 1)^2 - 42(2x + 1) + 49$

Guess and test factors of 9 with negative factors of 49 .

$$\begin{aligned} &= [3(2x + 1) - 7][3(2x + 1) - 7] \\ &= (6x + 3 - 7)(6x + 3 - 7) \\ &= (6x - 4)(6x - 4) \\ &= 4(3x - 2)^2 \end{aligned}$$

c) $(x^2 - x)^2 - 8(x^2 - x) + 12$

Find 2 numbers with sum -8 and product 12 .

$$\begin{aligned} &= [(x^2 - x) - 2][(x^2 - x) - 6] \\ &= (x^2 - x - 2)(x^2 - x - 6) \\ &= (x - 2)(x + 1)(x - 3)(x + 2) \end{aligned}$$

d) $(4x^2 + 4x + 3)^2 - 8(4x^2 + 4x + 3) + 12$

Find 2 numbers with sum -8 and product 12 .

$$\begin{aligned} &= [(4x^2 + 4x + 3) - 2][(4x^2 + 4x + 3) - 6] \\ &= (4x^2 + 4x + 3 - 2)(4x^2 + 4x + 3 - 6) \\ &= (4x^2 + 4x + 1)(4x^2 + 4x - 3) \\ &= (2x + 1)(2x + 1)(2x - 1)(2x + 3) \\ &= (2x + 1)^2(2x - 1)(2x + 3) \end{aligned}$$

C

17. Is $x + 1$ a factor of each polynomial expression?

a) $(x + 1)^2 + 4(x + 1) - 32$

Factor. Find 2 numbers with sum 4 and product -32 .

$$\begin{aligned} &= [(x + 1) + 8][(x + 1) - 4] \\ &= (x + 9)(x - 3) \end{aligned}$$

So, $x + 1$ is not a factor.

b) $2(2x + 3)^2 + 5(2x + 3) - 7$

Factor. Use guess and test with factors of 2 and factors of -7 .

$$\begin{aligned} &= [2(2x + 3) + 7][(2x + 3) - 1] \\ &= (4x + 13)(2x + 2) \\ &= 2(4x + 13)(x + 1) \end{aligned}$$

So, $x + 1$ is a factor.

18. Factor each polynomial.

a) $x^2 + x + \frac{1}{4}$

Find two numbers with sum 1 and product $\frac{1}{4}$.

$$= \left(x + \frac{1}{2}\right)\left(x + \frac{1}{2}\right), \text{ or } \left(x + \frac{1}{2}\right)^2$$

b) $x^2 + \frac{3}{2}x + \frac{1}{2}$

Find two numbers with sum $\frac{3}{2}$ and product $\frac{1}{2}$.

$$= (x + 1)\left(x + \frac{1}{2}\right)$$

c) $\frac{x^2}{2} - \frac{x}{4} - \frac{3}{4}$

$$= \frac{1}{2}\left(x^2 - \frac{1}{2}x - \frac{3}{2}\right)$$

Find two numbers with sum $-\frac{1}{2}$ and product $-\frac{3}{2}$.

$$= \frac{1}{2}(x + 1)\left(x - \frac{3}{2}\right)$$

d) $\frac{4}{3}x^2 - \frac{1}{12}y^2 = \frac{1}{3}\left(4x^2 - \frac{1}{4}y^2\right)$

$$= \frac{1}{3}\left(2x + \frac{1}{2}y\right)\left(2x - \frac{1}{2}y\right)$$

19. Factor each polynomial. Explain your strategy.

a) $x^2 - 6x + 9 - y^2$

The first 3 terms are a perfect square trinomial.

$$= (x - 3)^2 - y^2$$

Use the difference of squares.

$$= [(x - 3) + y][(x - 3) - y]$$

$$= (x + y - 3)(x - y - 3)$$

b) $4x^2 + 4x + 1 - 9y^2$

The first 3 terms are a perfect square trinomial.

$$= (2x + 1)^2 - 9y^2$$

Use the difference of squares.

$$= [(2x + 1) + 3y][(2x + 1) - 3y]$$

$$= (2x + 3y + 1)(2x - 3y + 1)$$

20. Consider the polynomial $3x^2 + nx - 4$. Determine a value for n so that $3x - 2$ is a factor of the polynomial.

$$\text{Write } 3x^2 + nx - 4 = (3x - 2)(x + a)$$

Equate the constant terms.

$$\text{If } 3x - 2 \text{ is a factor, then } -4 = -2a, \text{ so } a = 2$$

Substitute this value for a in the right side above, then expand.

$$(3x - 2)(x + 2) = 3x^2 + 4x - 4$$

The value of n is 4.