Lesson 3.4 Exercises, pages 217–226

A

4. Identify the values of a, b, and c to make each quadratic equation match the general form $ax^2 + bx + c = 0$.

a) $x^2 + 9x - 2 = 0$

b) $4x^2 - 11x = 0$

Compare each equation to $ax^2 + bx + c = 0$

a) $a = 1, b = 9, c = -2$

b) $a = 4, b = -11, c = 0$

c) $11x - 3x^2 + 8 = 0$

d) $3.2x^2 + 6.1 = 0$

c) $-3x^2 + 11x + 8 = 0$

$a = -3, b = 11, c = 8$

d) $a = 3.2, b = 0, c = 6.1$

5. Simplify each radical expression.

a) $\frac{6 \pm \sqrt{16}}{2} = \frac{6 \pm 4}{2}$

$\frac{6 + 4}{2} = 5$; or $\frac{6 - 4}{2} = 1$

b) $\frac{-8 \pm \sqrt{80}}{4} = \frac{-8 \pm 4\sqrt{5}}{4}$

$= \frac{4(-2 \pm \sqrt{5})}{4}$

$= -2 \pm \sqrt{5}$

c) $\frac{3 \pm \sqrt{45}}{6} = \frac{3 \pm 3\sqrt{5}}{6}$

$= \frac{3(1 \pm \sqrt{5})}{6}$

$= \frac{1 \pm \sqrt{5}}{2}$

d) $\frac{12 \pm \sqrt{28}}{4} = \frac{12 \pm 2\sqrt{7}}{4}$

$= \frac{2(6 \pm \sqrt{7})}{4}$

$= \frac{6 \pm \sqrt{7}}{2}$

B

6. Solve each quadratic equation.

a) $x^2 + 6x + 4 = 0$

b) $x^2 - 10x + 17 = 0$

For each equation, substitute for *a*, *b*, and *c* in:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

a) $a = 1, b = -6, c = 4$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(4)}}{2(1)}$

$x = \frac{6 \pm \sqrt{20}}{2}$

$x = \frac{6 \pm 2\sqrt{5}}{2}$

$x = 3 \pm \sqrt{5}$

b) $a = 1, b = -10, c = 17$

$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(17)}}{2(1)}$

$x = \frac{10 \pm \sqrt{32}}{2}$

$x = \frac{10 \pm 4\sqrt{2}}{2}$

$x = 5 \pm 2\sqrt{2}$

c) $x^2 + 4x - 3 = 0$

$a = 1, b = 4, c = -3$

$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-3)}}{2(1)}$

$x = \frac{-4 \pm \sqrt{28}}{2}$

$x = \frac{-4 \pm 2\sqrt{7}}{2}$

$x = -2 \pm \sqrt{7}$

d) $2x^2 - 2x - 1 = 0$

$a = 2, b = -2, c = -1$

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(2)(-1)}}{2(2)}$

$x = \frac{2 \pm \sqrt{12}}{4}$

$x = \frac{2 \pm 2\sqrt{3}}{4}$

$x = \frac{1 \pm \sqrt{3}}{2}$

7. Solve each quadratic equation.

a) $3x^2 = 4x + 1$

b) $4x^2 - 1 = -7x$

For each equation, substitute for a, b, and c in:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a)

$$3x^2 - 4x - 1 = 0$$
$$a = 3, b = -4, c = -1$$
$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(3)(-1)}}{2(3)}$$
$$x = \frac{4 \pm \sqrt{28}}{6}$$
$$x = \frac{4 \pm 2\sqrt{7}}{6}$$
$$x = \frac{2 \pm \sqrt{7}}{3}$$

b)

$$4x^2 + 7x - 1 = 0$$
$$a = 4, b = 7, c = -1$$
$$x = \frac{-7 \pm \sqrt{7^2 - 4(4)(-1)}}{2(4)}$$
$$x = \frac{-7 \pm \sqrt{65}}{8}$$

c) $2x(x - 3) = 4(x - 3) + 1$

$$2x^2 - 6x - 4x + 12 - 1 = 0$$
$$2x^2 - 10x + 11 = 0$$
$$a = 2, b = -10, c = 11$$
$$x = \frac{10 \pm \sqrt{(-10)^2 - 4(2)(11)}}{2(2)}$$
$$x = \frac{10 \pm \sqrt{12}}{4}$$
$$x = \frac{10 \pm 2\sqrt{3}}{4}$$
$$x = \frac{5 \pm \sqrt{3}}{2}$$

d) $(2x + 1)^2 + 2 = 0$

$$4x^2 + 4x + 1 + 2 = 0$$
$$4x^2 + 4x + 3 = 0$$
$$a = 4, b = 4, c = 3$$
$$x = \frac{-4 \pm \sqrt{4^2 - 4(4)(3)}}{2(4)}$$
$$x = \frac{-4 \pm \sqrt{-32}}{8}$$

The radicand is negative, so there are no real roots.

8. A student wrote the solution below to solve this quadratic equation:

$2x^2 - 3 = 7x$

$$x = -b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$
$$x = -(-7) \pm \frac{\sqrt{(-7)^2 - 4(2)(-3)}}{2(2)}$$
$$x = 7 \pm \frac{\sqrt{73}}{4}$$

Identify the error, then write the correct solution.

The student wrote an incorrect quadratic formula.
The correct solution is:

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4(2)(-3)}}{2(2)}$$
$$x = \frac{7 \pm \sqrt{73}}{4}$$

9. **a)** Solve each equation by factoring.

i) $3x^2 = 11x + 20$

$3x^2 - 11x - 20 = 0$

$(3x + 4)(x - 5) = 0$

$x = -\frac{4}{3}$ or $x = 5$

ii) $12x^2 + 8x = 15$

$12x^2 + 8x - 15 = 0$

$(2x + 3)(6x - 5) = 0$

$x = -\frac{3}{2}$ or $x = \frac{5}{6}$

b) Solve each equation in part a using the quadratic formula.

For each equation, substitute for a, b, and c in:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

i) $3x^2 - 11x - 20 = 0$

$a = 3, b = -11, c = -20$

$$x = \frac{11 \pm \sqrt{(-11)^2 - 4(3)(-20)}}{2(3)}$$

$$x = \frac{11 \pm \sqrt{361}}{6}$$

$$x = \frac{11 \pm 19}{6}$$

$$x = \frac{11 + 19}{6} = 5$$

Or, $x = \frac{11 - 19}{6} = -\frac{4}{3}$

ii) $12x^2 + 8x - 15 = 0$

$a = 12, b = 8, c = -15$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(12)(-15)}}{2(12)}$$

$$x = \frac{-8 \pm \sqrt{784}}{24}$$

$$x = \frac{-8 \pm 28}{24}$$

$$x = \frac{-8 + 28}{24} = \frac{5}{6}$$

Or, $x = \frac{-8 - 28}{24} = -\frac{3}{2}$

c) Which method do you prefer and why?

I prefer to factor when the numbers are small because it is quicker. I prefer to use the quadratic formula when the numbers are large and I have many factors to guess and test.

10. For each equation, choose a solution strategy, justify your choice, then solve the equation.

a) $2x^2 + 9x + 8 = 0$

I use the formula because I cannot factor.

Substitute: $a = 2, b = 9, c = 8$

in: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-9 \pm \sqrt{9^2 - 4(2)(8)}}{2(2)}$$

$$x = \frac{-9 \pm \sqrt{17}}{4}$$

b) $x^2 + 7x - 30 = 0$

I can factor.

$(x - 3)(x + 10) = 0$

$x = 3$ or $x = -10$

c) $(x + 6)^2 = 12$

$x^2 + 12x + 24 = 0$
I use the formula because I cannot factor. Substitute:
$a = 1, b = 12, c = 24$

in: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-12 \pm \sqrt{12^2 - 4(1)(24)}}{2(1)}$

$x = \dfrac{-12 \pm \sqrt{48}}{2}$

$x = \dfrac{-12 \pm 4\sqrt{3}}{2}$

$x = -6 \pm 2\sqrt{3}$

d) $8 + 5.6x - 1.2x^2 = 0$

I use the formula because I cannot factor. Substitute:
$a = -1.2, b = 5.6, c = 8$

in: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-5.6 \pm \sqrt{5.6^2 - 4(-1.2)(8)}}{2(-1.2)}$

$x = \dfrac{-5.6 \pm \sqrt{69.76}}{-2.4}$

$x = \dfrac{5.6 \pm \sqrt{69.76}}{2.4}$

11. Solve each quadratic equation. Give the solution to 3 decimal places.

a) $\frac{1}{3}x^2 - 3x + \frac{1}{4} = 0$

Multiply by 12.
$4x^2 - 36x + 3 = 0$
Substitute:
$a = 4, b = -36, c = 3$

in: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{36 \pm \sqrt{(-36)^2 - 4(4)(3)}}{2(4)}$

$x = \dfrac{36 \pm \sqrt{1248}}{8}$

$x = \dfrac{36 + \sqrt{1248}}{8}$

so, $x \doteq 8.916$

Or, $x = \dfrac{36 - \sqrt{1248}}{8}$

so, $x \doteq 0.084$

b) $-2x^2 + \frac{3}{2}x - \frac{4}{5} = 0$

Multiply by 10.
$-20x^2 + 15x - 8 = 0$
Substitute:
$a = -20, b = 15, c = -8$

in: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-15 \pm \sqrt{15^2 - 4(-20)(-8)}}{2(-20)}$

$x = \dfrac{-15 \pm \sqrt{-415}}{-40}$

There are no real roots.

c) $4.9x^2 + 12x - 0.8 = 0$

Substitute:
$a = 4.9, b = 12, c = -0.8$

in: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-12 \pm \sqrt{12^2 - 4(4.9)(-0.8)}}{2(4.9)}$

$x = \dfrac{-12 \pm \sqrt{159.68}}{9.8}$

$x = \dfrac{-12 + \sqrt{159.68}}{9.8}$

so, $x \doteq 0.065$

Or, $x = \dfrac{-12 - \sqrt{159.68}}{9.8}$

so, $x \doteq -2.514$

d) $2.1x^2 = 1.2x + 3$

Substitute:
$a = 2.1, b = -1.2, c = -3$

in: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{1.2 \pm \sqrt{(-1.2)^2 - 4(2.1)(-3)}}{2(2.1)}$

$x = \dfrac{1.2 \pm \sqrt{26.64}}{4.2}$

$x = \dfrac{1.2 + \sqrt{26.64}}{4.2}$

so, $x \doteq 1.515$

Or, $x = \dfrac{1.2 - \sqrt{26.64}}{4.2}$

so, $x \doteq -0.943$

12. Solve each radical equation. Check for extraneous roots.

a) $2 + \sqrt{5x} = 3x$

$$\sqrt{5x} = 3x - 2$$
$$(\sqrt{5x})^2 = (3x - 2)^2$$
$$5x = 9x^2 - 12x + 4$$
$$9x^2 - 17x + 4 = 0$$

Substitute:

$a = 9, b = -17, c = 4$

in: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{17 \pm \sqrt{(-17)^2 - 4(9)(4)}}{2(9)}$$
$$x = \frac{17 \pm \sqrt{145}}{18}$$

Use a calculator to check:

The root is: $x = \dfrac{17 + \sqrt{145}}{18}$

b) $2x = \sqrt{2x + 10} - 3$

$$2x + 3 = \sqrt{2x + 10}$$
$$(2x + 3)^2 = (\sqrt{2x + 10})^2$$
$$4x^2 + 12x + 9 = 2x + 10$$
$$4x^2 + 10x - 1 = 0$$

Substitute:

$a = 4, b = 10, c = -1$

in: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-10 \pm \sqrt{10^2 - 4(4)(-1)}}{2(4)}$$
$$x = \frac{-10 \pm \sqrt{116}}{8}$$
$$x = \frac{-10 \pm 2\sqrt{29}}{8}$$
$$x = \frac{-5 \pm \sqrt{29}}{4}$$

Use a calculator to check:

The root is: $x = \dfrac{-5 + \sqrt{29}}{4}$

13. a) Solve this equation using each strategy below: $x^2 - 10x - 24 = 0$

i) the quadratic formula

$$x^2 - 10x - 24 = 0$$

Substitute:

$a = 1, b = -10, c = -24$

in: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{10 \pm \sqrt{(-10)^2 - 4(1)(-24)}}{2(1)}$$
$$x = \frac{10 \pm \sqrt{196}}{2}$$
$$x = \frac{10 \pm 14}{2}$$
$$x = \frac{10 + 14}{2} = 12$$

Or, $x = \dfrac{10 - 14}{2} = -2$

ii) completing the square

$$x^2 - 10x - 24 = 0$$
$$x^2 - 10x = 24$$
$$x^2 - 10x + 25 = 24 + 25$$
$$(x - 5)^2 = 49$$
$$x - 5 = \pm\sqrt{49}$$
$$x = 5 \pm 7$$
$$x = 12 \text{ or } x = -2$$

iii) factoring

$$x^2 - 10x - 24 = 0$$
$$(x - 12)(x + 2) = 0$$
$$x = 12 \text{ or } x = -2$$

b) Which strategy do you prefer? Is it the most efficient? Explain.

Sample response: I prefer factoring; it is the most efficient because it takes less time and less space.

14. A person is standing on a bridge over a river. She throws a pebble upward. The height of the pebble above the river, h metres, is given by the formula $h = 26 + 9t - 4.9t^2$, where t is the time in seconds after the pebble is thrown.

a) When will the pebble be 20 m above the river? Give the answer to the nearest tenth of a second.

In $h = 26 + 9t - 4.9t^2$, substitute $h = 20$, then solve for t.

$$20 = 26 + 9t - 4.9t^2$$
$$0 = 6 + 9t - 4.9t^2$$

Substitute: $a = -4.9$, $b = 9$, $c = 6$ in: $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$t = \frac{-9 \pm \sqrt{9^2 - 4(-4.9)(6)}}{2(-4.9)}$$

$$t = \frac{9 \pm \sqrt{198.6}}{9.8}$$

Ignore the negative root since t cannot be negative.

$$t = \frac{9 + \sqrt{198.6}}{9.8}$$

$$t = 2.3563...$$

The pebble is 20 m above the river after approximately 2.4 s.

b) When will the pebble be 30 m above the river? Give the answer to the nearest tenth of a second.

In $h = 26 + 9t - 4.9t^2$, substitute $h = 30$, then solve for t.

$$30 = 26 + 9t - 4.9t^2$$
$$0 = -4 + 9t - 4.9t^2$$

Substitute: $a = -4.9$, $b = 9$, $c = -4$ in: $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$t = \frac{-9 \pm \sqrt{9^2 - 4(-4.9)(-4)}}{2(-4.9)}$$

$$t = \frac{9 \pm \sqrt{2.6}}{9.8}$$

$$t = \frac{9 + \sqrt{2.6}}{9.8}, \text{ or } 1.0829...$$

$$t = \frac{9 - \sqrt{2.6}}{9.8}, \text{ or } 0.7538...$$

The pebble is 30 m above the river after approximately 0.8 s and 1.1 s.

c) Why are there two answers for part b, but only one answer for part a?

There are two answers for part b because the stone is 30 m above the river on its way up and on its way down. There is only one answer for part a because the stone is only 20 m above the river on its way down.

15. A car was travelling at a constant speed of 19 m/s, then accelerated for 10 s. The distance travelled during this time, d metres, is given by the formula $d = 19t + 0.7t^2$, where t is the time in seconds since the acceleration began. How long did it take the car to travel 200 m? Give the answer to the nearest tenth of a second.

In $d = 19t + 0.7t^2$, substitute $d = 200$, then solve for t.

$200 = 19t + 0.7t^2$

$0 = -200 + 19t + 0.7t^2$

Substitute: $a = 0.7$, $b = 19$, $c = -200$ **in:** $t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$t = \frac{-19 \pm \sqrt{19^2 - 4(0.7)(-200)}}{2(0.7)}$$

$$t = \frac{-19 \pm \sqrt{921}}{1.4}$$

Ignore the negative root since t cannot be negative.

$$t = \frac{-19 + \sqrt{921}}{1.4}$$

$t = 8.1057...$

The car travelled 200 m in approximately 8.1 s.

16. Josie's rectangular garden measures 9 m by 13 m. She wants to double the area of her garden by adding equal lengths to both dimensions. Determine this length to the nearest centimetre.

Let the length added be x metres.

The new width, in metres, is: $x + 9$

The new length, in metres, is: $x + 13$

The new area, in square metres is: $(x + 9)(x + 13)$

The original area is: $(9)(13)$, **or** $117\ m^2$

The new area is: $2(117\ m^2) = 234\ m^2$

An equation is: $(x + 9)(x + 13) = 234$

$x^2 + 22x + 117 - 234 = 0$

$x^2 + 22x - 117 = 0$

Substitute: $a = 1$, $b = 22$, $c = -117$ **in:** $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-22 \pm \sqrt{22^2 - 4(1)(-117)}}{2(1)}$$

$$x = \frac{-22 \pm \sqrt{952}}{2}$$

Ignore the negative root since x cannot be negative.

$$x = \frac{-22 + \sqrt{952}}{2}$$

$x = 4.4272...$

The length added is approximately 4.43 m.

C

17. **a)** Solve this equation $\frac{1}{2}x^2 - \frac{3}{4}x - 1 = 0$ in the two ways described below:

i) Substitute the given coefficients and constant in the quadratic formula.

$\frac{1}{2}x^2 - \frac{3}{4}x - 1 = 0$

Substitute: $a = \frac{1}{2}$, $b = -\frac{3}{4}$, $c = -1$ in:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{\frac{3}{4} \pm \sqrt{\left(-\frac{3}{4}\right)^2 - 4\left(\frac{1}{2}\right)(-1)}}{2\left(\frac{1}{2}\right)}$$

$$x = \frac{3}{4} \pm \sqrt{\frac{41}{16}}$$

$$x = \frac{3}{4} \pm \frac{\sqrt{41}}{4}$$

ii) Multiply the equation by a common denominator to remove the fractions, then substitute in the quadratic formula.

$\frac{1}{2}x^2 - \frac{3}{4}x - 1 = 0$ Multiply by 4.

$2x^2 - 3x - 4 = 0$

Substitute: $a = 2$, $b = -3$, $c = -4$ in: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(-4)}}{2(2)}$$

$$x = \frac{3 \pm \sqrt{41}}{4}$$

b) Which strategy in part a do you prefer? Explain why.

I prefer the strategy in part ii because it is easier to work with integers than fractions.

18. This quadratic equation has only one root: $2x^2 + 6x + d = 0$
Use the quadratic formula to determine the value of d. Explain your strategy.

$2x^2 + 6x + d = 0$

Substitute: $a = 2$, $b = 6$, $c = d$ in: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(2)(d)}}{2(2)}$$

The equation has only one root, so the radicand must be 0.

$36 - 8d = 0$

$d = 4.5$

19. **a)** Solve this quadratic equation by expanding, simplifying, then applying the quadratic formula: $2(x - 5)^2 - 7(x - 5) - 2 = 0$

$$2x^2 - 20x + 50 - 7x + 35 - 2 = 0$$
$$2x^2 - 27x + 83 = 0$$

Substitute: $a = 2, b = -27, c = 83$ in: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{27 \pm \sqrt{(-27)^2 - 4(2)(83)}}{2(2)}$$

$$x = \frac{27 \pm \sqrt{65}}{4}$$

b) Solve the equation in part a using the quadratic formula without expanding.

$$2(x - 5)^2 - 7(x - 5) - 2 = 0$$

Substitute: $a = 2, b = -7, c = -2$ in: $x - 5 = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x - 5 = \frac{7 \pm \sqrt{(-7)^2 - 4(2)(-2)}}{4}$$

$$x - 5 = \frac{7 \pm \sqrt{65}}{4}$$

$$x = 5 + \frac{7 \pm \sqrt{65}}{4}$$

$$x = \frac{27 \pm \sqrt{65}}{4}$$

20. **a)** Is this equation quadratic: $x^4 + x^2 = 1$? Justify your response.

The equation is not quadratic because it contains an x^4-term.

b) Describe a strategy you could use to solve the equation in part a.

Write the equation as: $(x^2)^2 + (x^2) - 1 = 0$, then use the quadratic formula.

c) Solve the equation in part a.

Substitute: $a = 1, b = 1, c = -1$ in: $x^2 = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x^2 = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)}$$

$x^2 = \dfrac{-1 \pm \sqrt{5}}{2}$ Since x^2 is positive, ignore the negative root.

$$x = \pm\sqrt{\frac{-1 + \sqrt{5}}{2}}$$