Lesson 3.5 Exercises, pages 232–238

A

4. Calculate the value of the discriminant for each quadratic equation.

a) $5x^2 - 9x + 4 = 0$

In $b^2 - 4ac$, substitute:
$a = 5, b = -9, c = 4$
$b^2 - 4ac = (-9)^2 - 4(5)(4)$
$= 1$

b) $3x^2 + 7x - 2 = 0$

In $b^2 - 4ac$, substitute:
$a = 3, b = 7, c = -2$
$b^2 - 4ac = (7)^2 - 4(3)(-2)$
$= 73$

c) $18x^2 - 12x = 0$

In $b^2 - 4ac$, substitute:
$a = 18, b = -12, c = 0$
$b^2 - 4ac = (-12)^2 - 4(18)(0)$
$= 144$

d) $6x^2 + 7 = 0$

In $b^2 - 4ac$, substitute:
$a = 6, b = 0, c = 7$
$b^2 - 4ac = (0)^2 - 4(6)(7)$
$= -168$

5. The values of the discriminant for some quadratic equations are given. How many roots does each equation have?

a) $b^2 - 4ac = 36$

The discriminant is positive, so there are 2 real roots.

b) $b^2 - 4ac = 80$

The discriminant is positive, so there are 2 real roots.

c) $b^2 - 4ac = 0$

The discriminant is 0, so there is 1 real root.

d) $b^2 - 4ac = -4$

The discriminant is negative, so there are no real roots.

6. The values of the discriminant for some quadratic equations are given, where *a*, *b*, and *c* are integers. In each case, are the roots rational or irrational and can the equation be solved by factoring?

a) $b^2 - 4ac = 45$

The square root of the discriminant is irrational, so the roots are irrational and the equation cannot be solved by factoring.

b) $b^2 - 4ac = -16$

The square root of the discriminant is not a real number, so there are no real roots.

c) $b^2 - 4ac = 100$

The square root of the discriminant is rational, so the roots are rational and the equation can be solved by factoring.

d) $b^2 - 4ac = 0$

The discriminant is 0, so the root is rational and the equation can be solved by factoring.

B

7. Without solving each equation, determine whether it has one, two, or no real roots. Justify your answer.

a) $2x^2 - 9x + 4 = 0$

In $b^2 - 4ac$, substitute: $a = 2, b = -9, c = 4$

$$b^2 - 4ac = (-9)^2 - 4(2)(4)$$
$$= 49$$

Since $b^2 - 4ac > 0$, the equation has 2 real roots.

b) $-x^2 - 7x + 5 = 0$

In $b^2 - 4ac$, substitute: $a = -1$, $b = -7$, $c = 5$
$b^2 - 4ac = (-7)^2 - 4(-1)(5)$
$= 69$
Since $b^2 - 4ac > 0$, the equation has 2 real roots.

c) $2x^2 + 16x + 32 = 0$

In $b^2 - 4ac$, substitute: $a = 2$, $b = 16$, $c = 32$
$b^2 - 4ac = 16^2 - 4(2)(32)$
$= 0$
Since $b^2 - 4ac = 0$, the equation has 1 real root.

d) $2.55x^2 - 1.4x - 0.2 = 0$

In $b^2 - 4ac$, substitute: $a = 2.55$, $b = -1.4$, $c = -0.2$
$b^2 - 4ac = (-1.4)^2 - 4(2.55)(-0.2)$
$= 4$
Since $b^2 - 4ac > 0$, the equation has 2 real roots.

8. Determine the values of k for which each equation has two real roots, then write a possible equation.

a) $kx^2 + 6x - 1 = 0$ **b)** $6x^2 - 3x + k = 0$

For an equation to have 2 real roots, $b^2 - 4ac > 0$

a)
$a = k$, $b = 6$, $c = -1$
So, $6^2 - 4(k)(-1) > 0$
$4k > -36$
$k > -9$
Sample response:
$10x^2 + 6x - 1 = 0$

b)
$a = 6$, $b = -3$, $c = k$
So, $(-3)^2 - 4(6)(k) > 0$
$24k < 9$
$k < \frac{9}{24}$, or $\frac{3}{8}$
Sample response:
$6x^2 - 3x - 1 = 0$

9. Determine the values of k for which each equation has exactly one real root, then write a possible equation.

a) $2x^2 - kx + 18 = 0$ **b)** $kx^2 - 10x - 3 = 0$

For an equation to have exactly 1 real root, $b^2 - 4ac = 0$

a)
$a = 2$, $b = -k$, $c = 18$
So,
$(-k)^2 - 4(2)(18) = 0$
$k^2 = 144$
$k = \pm 12$
Sample response:
$2x^2 - 12x + 18 = 0$
or, $x^2 - 6x + 9 = 0$

b)
$a = k$, $b = -10$, $c = -3$
So,
$(-10)^2 - 4(k)(-3) = 0$
$12k = -100$
$k = -\frac{100}{12}$, or $-\frac{25}{3}$
Sample response:
$-\frac{25}{3}x^2 - 10x - 3 = 0$
or, $25x^2 + 30x + 9 = 0$

10. Determine the values of k for which each equation has no real roots, then write a possible equation.

a) $kx^2 - 9x - 3 = 0$

For an equation to have no real roots, $b^2 - 4ac < 0$
Substitute: $a = k$, $b = -9$, $c = -3$
$$(-9)^2 - 4(k)(-3) < 0$$
$$12k < -81$$
$$k < -\frac{81}{12}, \text{ or } -\frac{27}{4}$$
Sample response: $-10x^2 - 9x - 3 = 0$

b) $7x^2 - 6x + k = 0$

In $b^2 - 4ac < 0$, substitute: $a = 7$, $b = -6$, $c = k$
$$(-6)^2 - 4(7)(k) < 0$$
$$28k > 36$$
$$k > \frac{36}{28}, \text{ or } \frac{9}{7}$$
Sample response: $7x^2 - 6x + 2 = 0$

11. Can each equation be solved by factoring? If your answer is yes, solve it by factoring. If your answer is no, solve it using a different strategy.

a) $7x^2 - 8x - 12 = 0$

An equation factors if its discriminant is a perfect square.
In $b^2 - 4ac$, substitute: $a = 7$, $b = -8$, $c = -12$
$$b^2 - 4ac = (-8)^2 - 4(7)(-12)$$
$$= 400$$ **This is a perfect square.**
The equation can be solved by factoring.
$$7x^2 - 8x - 12 = 0$$
$$(7x + 6)(x - 2) = 0$$
$$x = -\frac{6}{7} \text{ or } x = 2$$

b) $14x^2 - 63x - 70 = 0$

Divide by 7.
$$2x^2 - 9x - 10 = 0$$
In $b^2 - 4ac$, substitute: $a = 2$, $b = -9$, $c = -10$
$$b^2 - 4ac = (-9)^2 - 4(2)(-10)$$
$$= 161$$ **This is not a perfect square.**
The equation cannot be solved by factoring.

Substitute: $a = 2$, $b = -9$ in: $x = \frac{-b \pm \sqrt{161}}{2a}$
$$x = \frac{9 \pm \sqrt{161}}{2(2)}$$
$$x = \frac{9 \pm \sqrt{161}}{4}$$

12. For each equation below:

i) Determine the value of the discriminant.

ii) Use the value of the discriminant to choose a solution strategy, then solve the equation.

a) $2x^2 - 6x + 1 = 0$

i) In $b^2 - 4ac$, substitute:
$a = 2, b = -6, c = 1$
$b^2 - 4ac = (-6)^2 - 4(2)(1)$
$= 28$

ii) This is not a perfect square, so use the quadratic formula.
Substitute: $a = 2, b = -6$ in:

$$x = \frac{-b \pm \sqrt{28}}{2a}$$

$$x = \frac{6 \pm \sqrt{28}}{2(2)}$$

$$x = \frac{3 \pm \sqrt{7}}{2}$$

b) $8x^2 - 3x - 5 = 0$

i) In $b^2 - 4ac$, substitute:
$a = 8, b = -3, c = -5$
$b^2 - 4ac = (-3)^2 - 4(8)(-5)$
$= 169$

ii) This is a perfect square, so use factoring.
$8x^2 - 3x - 5 = 0$

$(8x + 5)(x - 1) = 0$

$x = -\frac{5}{8}$ or $x = 1$

13. A model rocket is launched. Its height, h metres, after t seconds is described by the formula $h = 23t - 4.9t^2$. Without solving an equation, determine whether the rocket reaches each height.

a) 20 m

In $h = 23t - 4.9t^2$, substitute:
$h = 20$
$20 = 23t - 4.9t^2$
$0 = -20 + 23t - 4.9t^2$
If the rocket reaches a height of 20 m, then the equation has real roots.
In $b^2 - 4ac$, substitute:
$a = -4.9, b = 23, c = -20$
$b^2 - 4ac = 23^2 - 4(-4.9)(-20)$
$= 137$
Since the discriminant is positive, the equation has real roots, and the rocket reaches a height of 20 m.

b) 30 m

In $h = 23t - 4.9t^2$, substitute:
$h = 30$
$30 = 23t - 4.9t^2$
$0 = -30 + 23t - 4.9t^2$
If the rocket reaches a height of 30 m, then the equation has real roots.
In $b^2 - 4ac$, substitute:
$a = -4.9, b = 23, c = -30$
$b^2 - 4ac = 23^2 - 4(-4.9)(-30)$
$= -59$
Since the discriminant is negative, the equation has no real roots, and the rocket does not reach a height of 30 m.

14. Create three different quadratic equations whose discriminant is 64. Explain your strategy. What is true about these three equations?

Sample response:
Use guess and test to determine 3 values of a, b, and c so that:
$b^2 - 4ac = 64$ **Substitute:** $b = 0$
Then $-4ac = 64$**, which is satisfied by** $a = -4$ **and** $c = 4$
So, one equation is: $-4x^2 + 4 = 0$
$b^2 - 4ac = 64$ **Substitute:** $b = 4$
Then $16 - 4ac = 64$**, and** $-4ac = 48$**, which is satisfied by** $a = -3$ **and** $c = 4$
So, another equation is: $-3x^2 + 4x + 4 = 0$
$b^2 - 4ac = 64$ **Substitute:** $b = 2$
Then $4 - 4ac = 64$**, and** $-4ac = 60$**, which is satisfied by** $a = 5$ **and** $c = -3$
So, another equation is: $5x^2 + 2x - 3 = 0$
All 3 equations have 2 rational real roots.

C

15. Consider the equation $5x^2 + 6x + k = 0$. Determine two positive values of k for which this equation has two rational roots.

To be able to factor, the discriminant must be a perfect square.
In $b^2 - 4ac$**, substitute:** $a = 5$, $b = 6$, $c = k$
$6^2 - 4(5)(k) = 36 - 20k$
Use guess and test.

One perfect square is 16.
$$36 - 20k = 16$$
$$20k = 20$$
$$k = 1$$

Another perfect square is 4.
$$36 - 20k = 4$$
$$20k = 32$$
$$k = \frac{32}{20}\text{, or } 1.6$$

16. Create a quadratic equation so that a, b, and c are real numbers; the value of the discriminant is a perfect square; but the roots are not rational. Justify your solution by determining the value of the discriminant and the roots of the equation.

Sample response: The equation has the form $ax^2 + bx + c = 0$
Consider the quadratic formula:
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
If the roots are not rational, then a **or** b **is irrational.**
Use guess and test. Suppose $b = \sqrt{5}$.
Then $b^2 - 4ac$ **must be a perfect square.**
Substitute: $b = \sqrt{5}$
$5 - 4ac$ **must be a perfect square, such as 25.**
$$5 - 4ac = 25$$
$$4ac = -20$$
$ac = -5$**, which is satisfied by** $a = 1$, $c = -5$
An equation is: $x^2 + \sqrt{5}x - 5 = 0$
The discriminant is 25.
The roots are: $x = \frac{-\sqrt{5} \pm 5}{2}$

17. Consider the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

a) Write expressions for the two roots of the quadratic equation $ax^2 + bx + c = 0$.

The two roots are: $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ **and** $x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

b) Add the expressions. How is this sum related to the coefficients of the quadratic equation?

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-2b}{2a}, \text{ or } -\frac{b}{a}$$

The sum of the roots is the opposite of the quotient of the coefficient of x and the coefficient of x^2.

c) Multiply the expressions. How is this product related to the coefficients of the quadratic equation?

$$\left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right)\left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}\right)$$

$$= \frac{b^2 - (b^2 - 4ac)}{4a^2}$$

$$= \frac{4ac}{4a^2}, \text{ or } \frac{c}{a}$$

The product of the roots is the quotient of the constant term and the coefficient of x^2.

d) Use the results from parts b and c to write an equation whose roots are $x = -3 \pm \sqrt{11}$.

$$-\frac{b}{a} = (-3 + \sqrt{11}) + (-3 - \sqrt{11})$$

$$-\frac{b}{a} = -6, \text{ or } \frac{-6}{1}$$

$$\frac{c}{a} = (-3 + \sqrt{11})(-3 - \sqrt{11})$$

$$= 9 - 11$$

$$\frac{c}{a} = -2, \text{ or } \frac{-2}{1}$$

So, $a = 1$, $b = 6$, and $c = -2$

An equation is: $x^2 + 6x - 2 = 0$