

# REVIEW, pages 606–609

## 7.1

1. For each rational expression below:
- Identify the non-permissible values.
  - Write the expressions in simplest form.
  - Write an equivalent expression.

a)  $\frac{2x + 16}{x^2 - 64} = \frac{2(x + 8)}{(x + 8)(x - 8)}$

Non-permissible values:  $x = -8$  and  $x = 8$

$$\frac{\cancel{2(x+8)}}{\cancel{(x+8)}(x-8)} = \frac{2}{x-8}, x \neq -8, 8$$

$$\begin{aligned}\frac{2(x+8)}{(x+8)(x-8)} &= \frac{2(x+8)}{(x+8)(x-8)} \cdot \frac{x+8}{x+8} \\ &= \frac{2(x+8)^2}{(x+8)^2(x-8)}, x \neq -8, 8\end{aligned}$$

b)  $\frac{x^2 + x - 2}{-x^2 - 2x + 3} = \frac{(x+2)(x-1)}{-(x^2 + 2x - 3)}$

$$= \frac{(x+2)(x-1)}{-(x+3)(x-1)}$$

Non-permissible values:  $x = -3$  and  $x = 1$

$$\frac{(x+2)\cancel{(x-1)}}{-(x+3)\cancel{(x-1)}} = \frac{x+2}{-x-3}, x \neq -3, 1$$

$$\begin{aligned}\frac{(x+2)(x-1)}{-(x+3)(x-1)} &= \frac{(x+2)(x-1)}{-(x+3)(x-1)} \cdot \frac{x+2}{x+2} \\ &= \frac{(x+2)^2(x-1)}{-(x+3)(x-1)(x+2)}, x \neq -3, -2, 1\end{aligned}$$

## 7.2

2. Simplify each expression.

a)  $\frac{25ab^2}{4b} \cdot \frac{10b^3}{5a}$

$$\begin{aligned}&= \frac{\overset{5}{\cancel{25}} \cancel{a} b^{\cancel{2}}}{\underset{2}{\cancel{4}} \cancel{b}} \cdot \frac{\overset{10}{\cancel{10}} \cancel{b^3}}{\underset{5}{\cancel{5}} \cancel{a}} \\ &= \frac{25b^4}{2}, a \neq 0, b \neq 0\end{aligned}$$

b)  $\frac{6y^2}{y^2 - 9} \cdot \frac{y + 3}{15y}$

$$\begin{aligned}&= \frac{6y^2}{(y-3)(y+3)} \cdot \frac{y+3}{15y} \\ &= \frac{\overset{2}{\cancel{6}} y^{\cancel{2}}}{(y-3)\cancel{(y+3)}} \cdot \frac{\cancel{y+3}}{\underset{5}{\cancel{15}} y} \\ &= \frac{2y}{5(y-3)}, y \neq -3, 0, 3\end{aligned}$$

$$\begin{aligned}
 \text{c) } \frac{n-2}{n} \div \frac{3n-6}{n^2+n} &= \frac{n-2}{n} \div \frac{3(n-2)}{n(n+1)} \\
 &= \frac{n-2}{n} \cdot \frac{n(n+1)}{3(n-2)} \\
 &= \frac{n+1}{3}, n \neq -1, 0, 2
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } \frac{b^2+4b+3}{3-b} \div \frac{b^2+2b+1}{b^2-9} &= \frac{(b+1)(b+3)}{3-b} \div \frac{(b+1)(b+1)}{(b-3)(b+3)} \\
 &= \frac{(b+1)(b+3)}{3-b} \cdot \frac{(b-3)(b+3)}{(b+1)(b+1)} \\
 &= \frac{\cancel{(b+1)}(b+3)}{-(b-3)} \cdot \frac{\cancel{(b-3)}(b+3)}{\cancel{(b+1)}(b+1)} \\
 &= -\frac{(b+3)^2}{b+1}, b \neq -3, -1, 3
 \end{aligned}$$

### 7.3

3. Add or subtract.

$$\text{a) } \frac{6}{x^3} + \frac{5}{2x^4}$$

Common denominator:  $2x^4$

$$\begin{aligned}
 &= \frac{6}{x^3} \cdot \frac{2x}{2x} + \frac{5}{2x^4} \\
 &= \frac{12x}{2x^4} + \frac{5}{2x^4} \\
 &= \frac{12x+5}{2x^4}, x \neq 0
 \end{aligned}$$

$$\text{b) } \frac{-8}{3ef} - \frac{2}{4f} + \frac{5}{2e^2}$$

Common denominator:  $12e^2f$

$$\begin{aligned}
 &= \frac{-8}{3ef} \cdot \frac{4e}{4e} - \frac{2}{4f} \cdot \frac{3e^2}{3e^2} + \frac{5}{2e^2} \cdot \frac{6f}{6f} \\
 &= \frac{-32e}{12e^2f} - \frac{6e^2}{12e^2f} + \frac{30f}{12e^2f} \\
 &= \frac{-32e - 6e^2 + 30f}{12e^2f} \\
 &= \frac{2(-16e - 3e^2 + 15f)}{12e^2f} \\
 &= \frac{-16e - 3e^2 + 15f}{6e^2f}, e \neq 0, f \neq 0
 \end{aligned}$$

### 7.4

4. Add or subtract.

$$\text{a) } \frac{6}{v-2} + \frac{7}{2v+7}$$

Common denominator:  $(v-2)(2v+7)$

$$\begin{aligned}
 &= \frac{6}{(v-2)} \cdot \frac{(2v+7)}{(2v+7)} + \frac{7}{(2v+7)} \cdot \frac{(v-2)}{(v-2)} \\
 &= \frac{12v+42}{(v-2)(2v+7)} + \frac{7v-14}{(v-2)(2v+7)} \\
 &= \frac{19v+28}{(v-2)(2v+7)}, v \neq -\frac{7}{2}, 2
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } & \frac{2z}{3z^2 + 19z - 14} + \frac{3}{9z^2 - 12z + 4} \\
 &= \frac{2z}{(3z - 2)(z + 7)} + \frac{3}{(3z - 2)(3z - 2)} \\
 & \text{Common denominator: } (3z - 2)^2(z + 7) \\
 &= \frac{2z}{(3z - 2)(z + 7)} \cdot \frac{(3z - 2)}{(3z - 2)} + \frac{3}{(3z - 2)(3z - 2)} \cdot \frac{(z + 7)}{(z + 7)} \\
 &= \frac{6z^2 - 4z}{(3z - 2)^2(z + 7)} + \frac{3z + 21}{(3z - 2)^2(z + 7)} \\
 &= \frac{6z^2 - z + 21}{(3z - 2)^2(z + 7)}, z \neq -7, \frac{2}{3}
 \end{aligned}$$

## 7.5

5. Solve each equation.

$$\text{a) } \frac{3}{5f} = 1 - \frac{7}{2f}$$

Non-permissible value:  $f = 0$

Common denominator:  $10f$

$$\frac{3}{5f} = 1 - \frac{7}{2f}$$

$$\frac{3}{5f} = \frac{2f - 7}{2f}$$

$$2 \cdot 10f \left( \frac{3}{5f} \right) = 5 \cdot 10f \left( \frac{2f - 7}{2f} \right)$$

$$6 = 10f - 35$$

$$41 = 10f$$

$$f = \frac{41}{10}$$

$$\text{b) } \frac{4}{5x - 2} = \frac{3}{4x - 1}$$

Non-permissible values:

$$x = \frac{2}{5} \text{ and } x = \frac{1}{4}$$

Common denominator:

$$(5x - 2)(4x - 1)$$

$$\cancel{(5x - 2)}(4x - 1) \left( \frac{4}{\cancel{5x - 2}} \right)$$

$$= (5x - 2)\cancel{(4x - 1)} \left( \frac{3}{\cancel{4x - 1}} \right)$$

$$16x - 4 = 15x - 6$$

$$x = -2$$

$$\text{c) } \frac{2p}{p - 1} + \frac{p - 5}{p^2 - 1} = 1$$

$$\frac{2p}{p - 1} + \frac{p - 5}{(p - 1)(p + 1)} = 1$$

Non-permissible values:  $p = 1$  and  $p = -1$

Common denominator:  $(p - 1)(p + 1)$

$$\cancel{(p - 1)}(p + 1) \left( \frac{2p}{\cancel{p - 1}} \right) + \cancel{(p - 1)} \cancel{(p + 1)} \left( \frac{p - 5}{\cancel{(p - 1)} \cancel{(p + 1)}} \right) = (p - 1)(p + 1)(1)$$

$$2p^2 + 2p + p - 5 = p^2 - 1$$

$$p^2 + 3p - 4 = 0$$

$$(p + 4)(p - 1) = 0$$

$$p = -4 \text{ or } p = 1$$

$p = 1$  is a non-permissible value.

So, the only solution is  $p = -4$ .

**7.6**

6. Two fishing boats have the same average speed in still water. They leave a dock at the same time. One boat heads upstream and the other heads downstream. At a certain point, boat A is 56 km downstream and boat B is 24 km upstream. The average speed of the current is 8 km/h. What is the average speed of the boats in still water?

Let the average speed of the boats in still water be  $s$  km/h.

Average speed of current: 8 km/h

Boat A: downstream

Average speed downstream:  $(s + 8)$  km/h

Distance travelled: 56 km

Time for boat A:  $\frac{56}{s + 8}$  hours

Boat B: upstream

Average speed upstream:  $(s - 8)$  km/h

Distance travelled: 24 km

Time for boat B:  $\frac{24}{s - 8}$  hours

Boats take same time to travel these distances.

So, an equation is:  $\frac{56}{s + 8} = \frac{24}{s - 8}$ ,  $s > 8$

Non-permissible values:  $s = -8$  and  $s = 8$

Common denominator:  $(s + 8)(s - 8)$

$$\frac{56}{s + 8} = \frac{24}{s - 8}$$

$$\cancel{(s + 8)}(s - 8)\left(\frac{56}{\cancel{s + 8}}\right) = (s + 8)\cancel{(s - 8)}\left(\frac{24}{\cancel{s - 8}}\right)$$

$$56s - 448 = 24s + 192$$

$$32s = 640$$

$$s = 20$$

The average speed of the boats in still water is 20 km/h.